The Macroeconomics of Trade Credit*

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Abstract

In most countries, suppliers of intermediate goods and services are also the main providers of short-term financing to firms. This paper studies the macroeconomic implications of these financial links. In our model, trade credit is the outcome of a long-term contract between firms linked in the production process, and it is sustained in equilibrium by reputation forces as customers lose the relationship with their suppliers in case of a default. These financial links give rise to a *credit multiplier*: suppliers can enforce repayment of these IOUs, and they can discount these bills with banks to obtain liquidity. This process can either dampen or amplify the output effects of financial shocks, depending on the borrowing capacity of suppliers. Using Italian data, we find that the credit multiplier is sizable and show that trade credit substantially amplified the output costs of the Great Recession.

Keywords: Trade credit, financial crises, credit multiplier.

JEL codes: E44, G01

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1 Introduction

After the Great Recession, a significant amount of research was dedicated to determining the role of financial factors in overall economic instability. Most of the models used by economists to understand the macroeconomic implications of financial shocks—for instance, those building on the pioneering work of Kiyotaki and Moore (1997b) and Bernanke, Gertler, and Gilchrist (1999)—put capital markets and financial intermediaries at the forefront, by assuming that companies' funding needs are solely met by these institutions.

In reality, however, most companies around the world address the bulk of their liquidity needs using trade credit—the financing that suppliers of intermediate inputs provide in the form of extended payment terms. Not only trade credit is in most countries typically as large (and in many countries even larger) than short term loans and bonds issued by non-financial firms combined; but it is often also more volatile than these other debt instruments, underscoring the importance of understanding its drivers and the economic effects of these movements.

In this study, we propose a model for understanding the coexistence of bank and trade credit and quantifying the macroeconomic significance of this phenomenon. In our framework, firms can borrow from banks and from their suppliers of intermediate inputs. Unlike bank debt, which is partially upheld by law, trade credit relies on a reputation mechanism for enforcement, with customers having an incentive to repay it to avoid being cut off from future supplies of the goods. We show that trade credit works as a *credit multiplier*. On the one hand, firms that are financially constrained with banks can borrow from their suppliers, as they will have an incentive to repay that extra dollar in order to maintain those trade relationships. On the other hand, the suppliers can discount that extra dollar received with banks and obtain liquid funds to pay for their working capital. We show that this process increases the amount of credit flowing to firms and it reduces the economic distortions due to financial frictions. However, it can also make the supply chain more exposed to financial disruptions: when calibrating our model to Italian data, we show that the presence of trade credit substantially amplified the output costs of the Great Recession.

We consider an economy in which households consume a basket of differentiated goods. The production of these goods is organized on vertical supply chains (or sectors), with upstream firms producing intermediate goods using labor and downstream firms using these inputs to produce consumption goods. We follow Bigio and La'o (2020) and assume that part of the revenues of the final good firms are realized only after they purchase inputs, so these firms need credit to operate. Banks can lend to firms, but only up to a fraction of the firms' revenues. As in Jermann and Quadrini (2012), this debt limit is time-varying and stochastic, and we will think about an unexpected tightening as a negative credit supply shock. Importantly, we allow downstream firms to borrow also from their suppliers within the context of a long-term contract. These contracts specify not only the quantity of the good supplied and its price but also the financing—whether payments occur at the beginning or at the end of the period. We refer to the payments that occur at the end of the period as trade credit.

Trade credit emerges in equilibrium as part of the optimal contract between downstream and upstream firms. The enforcement of these credit relationships is sustained by a reputation mechanism, with the supplier excluding the customer from future provisions of the intermediate good in case of a default on the trade credit bill. So, upstream firms can supply more trade credit when the value of their relationship with the downstream firms is high, as in those cases, the latter have more incentives to repay. In our model, this value is endogenous, and it depends on characteristics of the supplier and the customer. For example, the value of the relationship is high when it is costly for the downstream firm to substitute a given intermediate input. Equilibrium trade credit is also affected by factors that govern the demand for funds by downstream firms. We allow some of these characteristics that affect the demand and supply of trade credit to differ across sectors in order to obtain a rich set of testable cross-sectional predictions.

We use this framework to study the macroeconomic implications of trade credit. To do so, we compare the behavior of our economy to that of a counterfactual "spot economy", which is identical in all respects to the former, with the exception that all economic entities engage in spot transactions. Focusing on a special case of the model that is analytically tractable, we present two sets of results. First we show that trade credit reduces the economic distortions due to financial frictions and brings the economy closer to the first best.[1] Second, we show that the presence of trade credit can make the economy more sensitive to financial shocks, amplifying their effects on output.

To understand the first result, let us consider the spot economy. Here, downstream firms need credit at the beginning of the period to purchase intermediate inputs from their suppliers, who then use these resources to pay for workers and to collect rents. In this context, financial frictions lower the amount of credit that goes to final good firms and, ultimately, they reduce what the economy can direct to remunerate the productive input (labor). This has the effect of decreasing the overall output produced.

In the economy with trade credit, instead, the suppliers can be paid also at the end of the period. This has two main effects. First, a larger share of the payments made by downstream firms in the morning can be directed toward the payment of workers, as trade

[1] By "first best", we mean the allocation achieved when credit markets are frictionless.

credit allows the suppliers to shift the payments of their rents to the end of the period.[2] Second, the suppliers can discount their accounts receivable (the IOUs offered to their customers) with banks and obtain liquid funds in the morning. This effectively allows the economy to multiply the amount of credit available to finance working capital, partly overcoming the distortions induced by financial frictions.

We then move on to study how trade credit relationships affect the propagation of financial shocks. In the spot economy, a tightening of firms' borrowing constraint decreases output. In the trade credit economy, the effects of the same shock are more subtle. When the suppliers have untapped borrowing capacity, they will respond by providing trade credit to their costumer, a force that dampens the effects of the financial shock on bank credit and output.[3] When the suppliers have a binding borrowing constraint with banks, the shock can have more detrimental effects relative to the spot economy: as suppliers need to cut on their issuance of trade credit, they reverse the credit multiplier process described earlier, and this amplifies the effects of the shock on bank credit and output.

We quantify the model using Italian firm-level balance sheet data from the historical ORBIS dataset for the 2007-2015 period. We aggregate this dataset at the sectoral level and collect data on the size of trade credit claims as well as other factors that in our model shape the cross-sectoral heterogeneity in trade credit: the share of intermediate inputs costs over sales, the accounts receivable of the sector, and the market concentration of suppliers.[4] In our theory, the first two factors determine the need for working capital while the third factor affects the value of relationships between upstream and downstream firms in that sector. Consistently, we find that sectors that have high working capital needs and that source intermediate inputs from more concentrated sectors obtain more trade credit from their suppliers. The calibrated model matches quantitatively these cross-sectional facts.

In addition, the model is consistent with evidence on the differential effects of financial shocks across the firms' distribution. In a dynamic difference in differences specification, we show that Italian firms that were tied to suppliers with tighter financial constraints experienced a substantially larger fall in trade credit and sales during the Great Recession. This finding mirrors a large body of empirical work, and it supports a key prediction of our theory: suppliers' ability to provide trade credit to their customers is key to understand

[2]This mechanism is closely linked to the "financial cost advantage of trade credit" explored in previous work by Garcia-Marin, Justel, and Schmidt-Eisenlohr (2019).

[3]By extending trade credit to their costumer and discounting these bills with banks, suppliers are effectively reducing the sensitivity of bank credit to the financial shock.

[4]To compute this object, we use our dataset to construct the sales Herfindahl-Hirschman index (HHI) for each sector in the economy and use sales share from the Italian input-output table to construct an HHI for the suppliers of a given sector.

how financial shocks propagate to the rest of the economy.[5]

We use the calibrated model to quantify the aggregate implications of trade credit for the Italian economy. For that purpose, we perform two exercises. First, to assess the importance of the credit multiplier, we compare the steady state of our economy with that of the spot economy. We find that bank credit in the former is much larger, and it allows the economy with trade credit to support 16% more output. Second, we study the response to financial shocks. We show that the economy with trade credit is twice as responsive as the spot economy to financial shocks of a similar depth and persistence as those observed during the Great Recession.

Literature. There is a large literature on trade credit in corporate finance. Researchers have documented that trade credit is one of the largest liability of firms across several countries. These loans have short maturity (typically between 30 and 90 days), they carry large implicit interest rates, and they are a fundamental tool used by firms to manage liquidity.[6] Several papers in this literature try to explain why we observe so much borrowing and lending between firms even in countries with well-developed financial markets. Most of them build on the hypothesis that suppliers of intermediate goods have some advantages over financial institutions when lending to their customers. In Biais and Gollier (1997) and Burkart and Ellingsen (2004), for example, suppliers can better monitor their customers, while in Frank and Maksimovic (1998), they can liquidate unused inputs more effectively because of their established network of buyers. The paper closest to ours in this literature is Cuñat (2007), who proposes a theory in which trade credit is supported in equilibrium by suppliers' threat of cutting off customers from all future provisions of the input.[7] The main contribution of our paper is to incorporate this idea in a business cycle model with aggregate shocks and study the macroeconomic implications of trade credit.

In this respect, we contribute to a large literature studying how financial shocks propagate to the rest of the economy. Jermann and Quadrini (2012) show that changes in the availability of short-term financing for firms are critical for understanding the macroeco-

[5]Starting from Meltzer (1960), several papers before ours have documented that suppliers balance sheet position have important implications on the costumer via a trade credit channel. Using US proprietary data, Costello (2020) finds that suppliers more exposed to liquidity shocks during the Great Recession reduce their trade credit supply to their customers relative to less exposed suppliers, which in turn had significant adverse effects on the performance of downstream firms. Other studies have documented substantial spillovers of suppliers' funding ability onto their customers' via a trade credit channel, see for instance Adelino, Ferreira, Giannetti, and Pires (2023), Giannetti, Serrano-Velarde, and Tarantino (2021) and Bottazzi, Gopalakrishna, and Tebaldi (2023) for recent papers.

[6]See Amberg, Jacobson, Von Schedvin, and Townsend (2021) for evidence on the role of trade credit as a liquidity management tool.

[7]See Brugues (2023) for a related theory and empirical evidence. See also Clayton, Maggiori, and Schreger (2023) for a related idea in the context of power relationships across countries.

nomic impact of financial shocks. Despite this finding, and the fact that in many countries trade credit is the largest form of short-term financing for firms, only recently researchers have started to focus on its macroeconomic implications.[8] Altinoglu (2021) and Luo (2020) introduce trade credit relationships in the production network economy of Bigio and La'o (2020) in order to study how financial shocks propagate. In those papers, unlike in ours, trade credit is exogenous and does not respond to aggregate shocks.

The work closest to ours in this literature is Reischer (2020), which builds on Altinoglu (2021) and Luo (2020) but allows the price and quantity of trade credit to respond to shocks. In her model, as in ours, trade credit can potentially dampen or amplify the effects of credit supply shocks. We view the two papers as complementary. While Reischer (2020) deals with a richer production network than we do, our framework microfounds trade credit and its relationship with bank finance, leading to novel insights. For example, the core mechanism in our model is the spillover of trade credit on the quantity of bank credit—which we label, the credit multiplier. In Reischer (2020), this interdependence is absent because bank credit is modeled via an exogenous rule.[9]

The concept of a credit multiplier is related to the seminal work of Holmstrom and Tirole (1997). In that paper, a moral hazard problem limits the amount of funds that investors can give to firms. Banks—who have access to a monitoring technology—can borrow from the first group and lend to the second, a process that increases overall credit available to firms in the economy. In our framework, credit is multiplied because suppliers can enforce repayment of trade credit *and* they discount their invoices with financial institutions. Invoice discounting has been historically a core function of the financial sector, and as we discuss in the paper it still represents a large fraction of the short-term credit provided to non-financial corporations in many countries.

Finally, our paper is related to studies that have introduced optimal contracts in general equilibrium models with aggregate shocks.[10] The paper closest to ours is Cooley, Marimon, and Quadrini (2004), which studies the implications of limited enforcement of financial contracts for the propagation of aggregate technological shocks. We instead focus on trade

[8] An exception is the early contribution of Kiyotaki and Moore (1997a), who develop a model with endogenous trade credit relationships and show that firm-specific shocks can be amplified through these credit chains.

[9] Another important difference is that trade credit in our framework is a forward-looking variable as it depends on the value of future relationships between firms, while in Reischer (2020) it is the outcome of a static decision. This leads to different predictions regarding the role of expectations in determining trade credit relationships and potentially different policy prescriptions when dealing with payments' crises.

[10] See, for instance, the work of Kehoe and Perri (2002), Dovis (2019), and Aguiar, Amador, and Gopinath (2009) for applications to international capital flows; Boldrin and Horvath (1995) and Souchier (2022) for the role of long-term wage contracts for labor market fluctuations; and Di Tella (2017), who studies how the optimal state contingency of financial contracts affects the financial amplification mechanism.

credit and its role for the amplification of financial shocks. From a technical point of view, our model features permanent sectoral heterogeneity, which makes the distribution of promised values a (high-dimensional) aggregate state variable. In the numerical algorithm, we deal with this issue by nesting the solution of the optimal contract within a fixed point problem à la Krusell and Smith (1998).

2 The model

We consider an economy populated by infinitely lived households, firms, and banks. Households supply labor and consume a bundle of imperfectly substitutable goods. These final goods are produced by a continuum of firms that combine capital with intermediate inputs, while the intermediate inputs are produced by monopolists using labor. We will refer to the production process for a specific final good as a supply chain or sector. There is a lag between production and the full receipt of payments, so final good firms need credit to pay for intermediate inputs. Credit is provided by competitive banks and by the suppliers of intermediate inputs. We describe the environment in detail in Section 2.1, define an equilibrium in Section 2.2, and discuss some of the simplifying assumptions we made in Section 2.3.

2.1 Environment

Time is discrete and indexed by $t = 0, 1, 2, \ldots$. Uncertainty is described by a Markov process that takes finite values in the set $\mathcal{S}$. We denote by s_t the state of the process at time t and by $s^t = (s_0, s_1, \ldots, s_t)$ the history of states up to period t. The process for s_t is given by the transition matrix $\pi(s_{t+1}|s_t)$. All equilibrium variables are in general functions of the history s^t, but whenever no confusion is possible, we leave this dependence implicit and only use a subscript t.

Households. Households supply labor to firms at the competitive wage W_t and every period receive the profits from firms in the economy, Π_t. They use this income to purchase a continuum of imperfectly substitutable final goods $\{y_{i,t}\}$ at prices $\{p_{i,t}\}$. So, the budget constraint of the representative household is given by

$$\int p_{i,t} y_{i,t} di = W_t L_t + \Pi_t.$$

Households choose labor and final goods to maximize their lifetime utility,

$$U = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[C_t - \chi \frac{L_t^{1+\psi}}{1+\psi} \right] \right\},$$

where β is the rate of time preference, ψ is the inverse Frisch elasticity of labor supply, and C_t is a CES aggregator of final goods:

$$C_t = \left(\int y_{i,t}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}.$$

This optimization problem yields two familiar optimality conditions, one for labor supply and one for the demand for final good i,

$$\chi L_t^{\psi} = W_t \tag{1}$$

$$y_{i,t} = \left(\frac{p_{i,t}}{P_t} \right)^{-\gamma} C_t, \tag{2}$$

where P_t is the price index for the consumption bundle C_t, and we normalize it to 1.

Production. The production of final goods is carried out by a continuum of competitive firms. These firms produce final goods using capital k and N_i imperfectly substitutable intermediate inputs. We denote by $\{x_{ij,t}\}$ the j-th variety of intermediate good used in the production of a final good of type i at date t. The technology to produce the final good is

$$y_{i,t} = k^{1-\eta_i} \left\{ \left[\sum_{j=1}^{N_i} x_{ij,t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\}^{\eta_i},$$

where σ is the elasticity of substitution across intermediate inputs and η_i governs the intermediate-input share of production. As in Atkeson and Burstein (2008), we assume that $\sigma > \gamma$, that is intermediate inputs are more substitutable than final goods.

The intermediate inputs $x_{ij,t}$ are produced by monopolists using a linear technology with labor as the sole production input,

$$x_{ij,t} = l_{ij,t}.$$

The sources of technological heterogeneity across different supply chains are twofold: the share of intermediate inputs in the production process—represented by η_i—and the degree of market concentration of upstream firms—represented by N_i.

Financial markets. Each period is split into two stages. In the first stage (the morning), final good firms receive the intermediate inputs, start the production process and obtain a fraction of their sales. In the second stage (the afternoon), final good firms finish production and receive the remainder of their sales. The lag between production and the receipt of sales is due to two factors. First, there is time to build: firms produce a fraction δ of output in the morning and the remainder in the afternoon.[11] Second, a fraction π_i of the morning sales is paid by households in the afternoon. Therefore, final good firms receive only a fraction $\delta(1-\pi_i)$ of their overall sales in the morning. Because of these assumptions, final good firms may need credit in the morning in order to purchase intermediate inputs. Credit can be obtained from two sources: competitive banks and the suppliers of intermediate inputs.

Competitive banks collect wages from households in the morning and offer loans to downstream and upstream firms. We denote by $b_i(s^t)$ the amount borrowed by final good firms that produce final good i at time t for history s^t. Similarly, $b_{ij}(s^t)$ denotes the amount borrowed by the intermediate good monopolist that sells variety j to final good firms of type i. As in Bocola and Lorenzoni (2022), firms cannot commit to repaying the debt in the afternoon, and if they default, they suffer a penalty equal to a fraction $1-\theta(s_t)$ of their afternoon revenues. Given these assumptions, final good firms have an incentive to repay their debt in the afternoon as long as

$$b_i(s^t) \leq [1-\theta(s_t)]\,[1-\delta(1-\pi_i)]\,rev_i(s^t), \tag{3}$$

where $rev_i(s^t) \equiv p_i(s^t)y_i(s^t)$ is the revenue of firm i in history s^t. As in Jermann and Quadrini (2012), we interpret $\theta(s_t)$ as a "financial shock": when $\theta(s_t)$ increases, final good firms will face a tighter borrowing constraint with banks, and this will potentially impact their production choices. These shocks are the only source of uncertainty in the model.

In addition to borrowing from banks, final good firms can borrow from their suppliers of intermediate goods. That is, when selling a good $x_{ij}(s^t)$ to a final good firm, the monopolist specifies a spot payment to be made in the morning, $p^{\mathrm{s}}_{ij}(s^t)$, and a payment to be made in the afternoon, $p^{\mathrm{tc}}_{ij}(s^t)$. Effectively, $p^{\mathrm{tc}}_{ij}(s^t)$ is the *trade credit* offered by the monopolist. We will discuss momentarily how the terms of the contract are determined.

Depending on the terms of the contract, the upstream producers may also need to borrow from banks. This happens when the payments they receive in the morning are smaller than their costs of production (the wages of workers), that is when $p^{\mathrm{s}}_{ij}(s^t) \leq W(s^t)x_{ij}(s^t)$. In that case, the upstream firm needs to borrow the difference from banks. When borrowing from banks, they face the same incentive problem described earlier for the final good firms,

[11] We follow most of the macro literature and assume that time to build is exogenous. See Antràs (2023) for a recent paper that endogenizes this feature in a general equilibrium model.

so their borrowing constraint with banks is,

$$b_{ij}(s^t) \leq [1-\theta(s_t)]p_{ij}^{\text{tc}}(s^t). \tag{4}$$

That is, upstream firms can discount a fraction $[1-\theta(s_t)]$ of their afternoon revenues (their accounts receivable $p_{ij}^{\text{tc}}(s^t)$), and use the proceeds to pay for their workers.

Trade credit. The trade credit contract is the outcome of a long-term relationship between upstream and downstream firms operating in the supply chain i. We assume that upstream firms make a take-it-or-leave-it offer to downstream firms, specifying the terms of the contract, $\{x_{ij}(s^t), p_{ij}^{\text{s}}(s^t), p_{ij}^{\text{tc}}(s^t)\}$ for all $\{s^t\}$. Differently from bank credit, firms that default on their trade credit do not suffer any direct loss of revenues. The enforcement of these contracts is instead guaranteed by a reputation mechanism: a final good firm that defaults at time t on its supplier is permanently excluded from purchasing that intermediate good from time $t+1$ onward.[12]

We denote by $J_i(s^t)$ the expected discounted value of a final good firm operating in sector i at time t under the contract,

$$J_i(s^t) = \sum_{\tau=0}^{\infty}\sum_{s^{t+\tau}} \beta^\tau \pi\left(s^{t+\tau}\middle| s^t\right)\left[rev_i(s^{t+\tau}) - \sum_{j=1}^{N_i}\left(p_{ij}^{\text{s}}(s^{t+\tau}) + p_{ij}^{\text{tc}}(s^{t+\tau})\right)\right],$$

where $\pi\left(s^{t+\tau}\middle| s^t\right) = \prod_{l=1}^{\tau}\pi\left(s_{t+l}|s^{t+l-1}\right)$ is the probability of arriving at history $s^{t+\tau}$ from s^t. We denote by $J_i^{(-j)}(s^t)$ the corresponding value when the firm cannot purchase intermediate inputs from firm j starting at period t. The *value of the relationship* with supplier j for a downstream firm is then given by $\tilde{J}_i^j(s^t) \equiv J_i(s^{t+1}) - J_i^{(-j)}(s^{t+1})$.

Since a permanent break in the trade relationship is the only cost of defaulting on trade credit, final good firms will have an incentive to repay their trade credit to their supplier as long as

$$p_{ij}^{\text{tc}}(s^t) \leq \beta\mathbb{E}\left[\tilde{J}_i^j(s^{t+1})|s^t\right]. \tag{5}$$

The constraint in (5) is a key condition in the model. It says that the greater the value of the relationship between the customer and the supplier of variety j, the more trade credit the supplier can extend.[13]

Each supplier j chooses the optimal contract to maximize the present discounted value

[12] The threat made by the supplier is credible given our assumption that there is a continuum of final good producers.

[13] In our economy trade credit is always repaid in equilibrium. This is an abstraction: in practice, firms do default on these claims, see Mateos-Planas and Seccia (2021) for an analysis of equilibrium default on trade credit.

of its profits,

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \pi \left(s^t \middle| s^t \right) \left[p^{\text{s}}_{ij}(s^t) + p^{\text{tc}}_{ij}(s^t) - W(s^t) x_{ij}(s^t) \right] \right\} \tag{6}$$

taking as given the wage $W(s^t)$, the consumers' demand for industry i goods in equation (2), the no-default constraints (3) and (4), the trade credit constraint (5), the participation constraint of the final good producer, $\tilde{J}^j_i(s^{t+1}) \geq 0 \ \forall s^{t+1}$, and the feasibility requirement that $rev_i(s^t) - p^{\text{s}}_{ij}(s^t) - p^{\text{s}}_{ij}(s^t) \geq 0 \ \forall s^t, j$.[14] The suppliers of intermediate goods take as given aggregate quantities and the trade credit contracts of all other suppliers but internalize how their trade credit contract affects $y_i(s^t)$ and $p_i(s^t)$.

2.2 Equilibrium

A symmetric equilibrium is a set of aggregate variables $\{L(S^t), C(s^t), W(s^t), \Pi(s^t)\}$ for all s^t, final good firm quantities and prices $\{y_i(s^t), p_i(s^t)\}$ for all i and s^t, and intermediate-input firm quantities and prices $\{x_i(s^t), p^{\text{s}}_i(s^t), p^{\text{tc}}_i(s^t)\}$ for all i and s^t, such that:[15]

1. Given aggregate prices and profits, aggregate consumption and labor solve the household's problem.
2. Given aggregate prices and quantities as well as the trade credit contracts of all other suppliers, the set $\{y_i(s^t), p_i(s^t), x_i(s^t), p^{\text{s}}_i(s^t), p^{\text{tc}}_i(s^t)\}$ solve the problem of the intermediate goods firm supplying to firm i.
3. The labor market clears for every history s^t:

$$L(s^t) = \int_i N_i x_i(s^t) di.$$

2.3 Discussion

Before moving on, let us discuss some of the key assumptions of our model.

First, our model features an asymmetry in the enforcement of bank and trade credit: a firm that defaults on a bank loan loses part of its revenues, a legal punishment that is

[14] In principle, we should also include the incentive constraints that allow for multiple deviations (e.g., final good firms defaulting on both banks and firms or defaulting on multiple firms at the same time). If final good firms were to default on banks and a supplier j, they would lose $[1-\theta(s_t)]\,[1-\delta(1-\pi_i)]\, rev_i(s^t)$ and the expected discounted surplus of the match with supplier j. So, if (3) and (5) are satisfied, there is no incentive to default on both. It is possible to show that in a deterministic steady state, the trade credit constraints (5) also imply that firms have no incentives to default on the other suppliers as long as $\sigma > \gamma$. This is also true outside the steady state in all our numerical experiments.

[15] We do not include a j subscript for intermediate-input goods because in a symmetric equilibrium, all suppliers to firm i will have the same quantities and prices.

not present when a firm defaults on trade credit. This assumption captures, in a stylized fashion, the fact that bank credit typically carries much better legal protections than trade credit, see Cuñat and García-Appendini (2012) and Jacobson and Von Schedvin (2015) for a discussion.[16]

The enforcement of trade credit in our model is instead supported by a reputation mechanism. There is ample empirical evidence consistent with this idea. Using detailed transaction data on a large US poultry exporter, Antras and Foley (2015) document that the intensity of trade credit to their costumers increases with the length of the relationship, a fact that has been confirmed and extended using rich transaction-level data by Benguria, Garcia-Marin, and Schmidt-Eisenlohr (2023). In our model, these financial links are supported by the fact that relationships between a supplier and a costumer are valuable and that they will be destroyed in the event a costumer defaults on the supplier. This mechanism is supported by a number of studies that have documented large economic costs when a costumer loses a link to a supplier (Barrot and Sauvagnat, 2016; Carvalho, Nirei, Saito, and Tahbaz-Salehi, 2021). In addition, Garcia-Marin, Justel, and Schmidt-Eisenlohr (2019) show that suppliers with larger markups provide more trade credit to their costumers, consistent with the idea that the higher the market power of the supplier, the more incentives for costumers to service trade credit and mantain a good relationship.

Second, in our framework credit flows only from suppliers to costumers, and not the other way around—something that the literature refers as a cash in advance contract.[17] It is straightforward to modify our framework to allow costumers to provide credit to their supplier, for example by introducing time to build also on the supplier side. In many applications, cash in advance contracts are much less common than trade credit contracts (Antras and Foley, 2015; Benguria, Garcia-Marin, and Schmidt-Eisenlohr, 2023), so we do not expect such a modification to materially impact the results.

Third, we assume that suppliers can commit to the entire path of quantities and prices offered. It is worth noting that this makes the decision problem of suppliers *time inconsistent*, as they have an incentive to promise higher quantities and lower prices in the future in order to extend more trade credit to their customer in the current period. Alternatively, we could have assumed that suppliers cannot commit to the entire path of future actions and solve for the Markov-perfect equilibrium. The key difference is that without commitment the supplier would take the trade credit limit as given, while in the current framework it

[16]The assumption of a static penalty when defaulting on banks is made for tractability. We could allow for a dynamic punishment by assuming that a firm that defaults on banks is temporarily excluded from financial markets as in Arellano (2008), which would make the debt limits (3) and (4) depend on future values. The key insights of our analysis would go through in this version of the model.

[17]See Bigio (2023) for a model that features this type of contracts and an endogenous network of payments.

has a motive to promise better terms in the future to its costumers when the trade credit constraint is binding. Aside from this difference, the two models would be identical.

Fourth, firms in our framework cannot accumulate cash holdings. In making this assumption, we stay close to the literature that studies the misallocation of production due to financial frictions in a multi-sector framework, see Bigio and La'o (2020) and the literature that followed. It is well known that allowing firms to accumulate cash holdings reduces the costs of financial constraints. However, the key qualitative insights of our paper survive in this setting as long as financial constraints are binding in the economy where firms can accumulate cash.

Finally, our analysis considers a closed economy. We do so to isolate the key novel mechanism of our model, the credit multiplier. It would be interesting to extend our framework to an open economy setting as most of international trade in goods is financed via trade credit (Schmidt-Eisenlohr, 2013; Auboin, 2009). In addition, the presence of multiple currencies opens up interesting channels of transmission. For example, Hardy and Saffie (2024) and Hardy, Saffie, and Simonovska (2023) focus on currency mismatches documenting that large firms in emerging markets provide trade credit in domestic currency to local producers and at the same time borrow in foreign currency from the rest of the world, engaging in a carry trade.

3 Macroeconomic implications of trade credit

We now study the macroeconomic implications of trade credit in our economy. For this purpose, we will discuss some key properties of the model—the deterministic steady state and the response of endogenous variables to a financial shock—and compare these predictions to those of an economy that is identical to the benchmark with the exception that intermediate good producers cannot extend trade credit to their customers, $p^{\text{tc}}(s^t) = 0$. We will refer to the latter as the *spot economy*.

We provide two main results. First, we show that trade credit relationships allow the economy to support more bank credit and to better allocate it toward productive uses, making it possible to sustain a higher level of output on average. This analysis is contained in Section 3.1. Second, we show in Section 3.2 that trade credit can either dampen or amplify the macroeconomic effects of financial shocks depending on the borrowing capacity of suppliers.

To make the analysis more transparent, we will consider a special case of our economy that is analytically tractable.[18] We focus on an economy with only one sector and one

[18]In the next section we discuss to what extent these results extend to the fully fledged model.

supplier of intermediate inputs, $N_i = 1$. We normalize the level of capital for final good firms to 1 so that the production function is $y = x^\eta$.[19] We further assume that $\pi_i = 0$, so final good firms receive a fraction δ of their revenues in the morning and the remainder in the afternoon. In addition, we set the inverse of the Frisch elasticity to zero, $\psi = 0$. This last assumption implies that the wage is constant over time and given by $W = \chi$. Given these assumptions, the equilibrium is fully characterized by solving the decision problem of the monopolist.

We start by studying the spot economy and later move on to the benchmark.

The spot economy. In the spot economy final good firms need to pay their suppliers in the morning. They can do that out of the cash they receive in the morning, $\delta x^\eta(s^t)$, or by borrowing from banks with a debt limit given by $[1-\theta(s_t)](1-\delta)x^\eta(s^t)$. This implies that the payment that final good firms make in the morning to the monopolist, $p^s(s^t)$, cannot be larger than $\{\delta + (1-\delta)[1-\theta(s_t)]\}\, x^\eta(s^t)$.

The monopolist chooses $\{x(s^t), p^s(s^t)\}$ to maximize the present discounted value of profits subject to the borrowing constraints of the final good firms and their participation constraints. Since $N_i = 1$, the participation constraints boil down to $J(s^{t+1}) \geq 0$ for all s^{t+1}. This problem is equivalent to solving a static profit maximization problem, which yields the first-order condition

$$[\delta + (1-\delta)(1-\theta)]\eta x^{\eta-1} = W. \tag{7}$$

To understand the behavior of the monopolist, suppose first that credit markets are frictionless, $\theta = 0$. In this case, the monopolist chooses labor to equate its marginal product to the wage, $\eta x^{\eta-1} = W$, and it extracts all the rents from the final good producers by setting $p^s = x^\eta$.[20] Incidentally, we can see that in this case there are no output distortions even though the supplier has monopoly power, because the latter can set prices non-linearly.[21] This solution is not feasible when $\theta > 0$. Due to the borrowing constraints, final good firms cannot borrow in the morning the entire revenue stream, and so the supplier cannot extract all the rents from the final good firms. This induces a "wedge"—equal to $[\delta + (1-\theta)(1-\delta)]$—between the marginal product of labor and the wage, distorting down the scale of production relative to the economy with no financial frictions.

Using equation (7), we can further characterize the response of labor to a financial shock

[19] In a slight abuse of notation, we do not index variables by the sector i and the identity of the supplier j as we did in Section 2 because there is only one sector and one supplier in this example.

[20] Because the intermediate good is produced one-for-one with labor, $\eta x^{\eta-1}$ is also the marginal product of labor.

[21] For a discussion of how non-linear pricing breaks the link between markups and misallocation, see Bornstein and Peter (2023).

in the spot economy as follows

$$\varepsilon_{x,\theta}^{\text{spot}} \equiv \frac{d \ln x^{\text{spot}}}{d \ln \theta} = -\frac{1}{1-\eta} \frac{\theta}{1-\theta} \alpha^{\text{spot}}, \tag{8}$$

where $\alpha^{\text{spot}} \equiv \frac{(1-\delta)(1-\theta)}{\delta+(1-\delta)(1-\theta)}$ is the share of bank credit over total funds available to final good firms in the morning, a notion of the importance of bank leverage for the operation of the supply chain.

When θ increases, final good firms can borrow less from banks, and this reduces their funds available to purchase intermediate goods in the morning. This leads to a reduction in the quantity of intermediate inputs produced and, ultimately, a reduction of output in the economy.

Importantly, the strength of this channel depends on the leverage of the supply chain—captured by the term α^{spot} in equation (8). This term ranges between 0 and 1, depending on the value of δ. When δ is close to one, final good firms receive most of the cashflows in the morning, so bank credit is not that important for financing the operations of the supply chain. In these cases, α^{spot} is close to zero, and financial shocks have limited effects on output. Conversely, when δ is close to zero, bank credit is essential for financing the purchase of intermediate inputs, so a financial shock has a larger impact on output.

The economy with trade credit. In the trade credit economy the monopolist has one additional instruments to maximize profits—offering their costumers to delay their payments in the afternoon up to the limit implied by the trade credit constraint (5). This constraint makes the decision problem of the monopolist dynamic, as future rents promised to costumers influence how much trade credit can be supported today. We can write this decision problem using recursive methods.

Let J be the rents that the monopolist has promised to a final good firm after history s^t, and let θ be the realization of the financial shock at s_t. The monopolist chooses labor x, the morning and afternoon payments $\{p^{\text{s}}, p^{\text{tc}}\}$, as well as the future continuation values for final good firms conditional on any realization of the financial shock next period, $J'(\theta')$, to maximize the present discounted value of profits,

$$V(J,\theta) = \max_{x,p^{\text{s}},p^{\text{tc}},J'(\theta')} \left(p^{\text{s}} + p^{\text{tc}} - Wx\right) + \beta \mathbb{E}\left[V(J'(\theta'),\theta')|J,\theta\right].$$

subject to the debt limits that final good firms and the monopolist face when borrowing from banks,

$$\begin{aligned} p^{\mathrm{s}} &\leq [\delta + (1-\delta)(1-\theta)]x^{\eta}, \quad (9) \\ Wx - p^{\mathrm{s}} &\leq (1-\theta)p^{\mathrm{tc}}, \quad (10) \end{aligned}$$

the debt limit that final good firms face when borrowing from the monopolist,

$$p^{\mathrm{tc}} \leq \beta \mathbb{E}[J'(\theta')|J,\theta], \quad (11)$$

the promise-keeping constraint,

$$J = x^{\eta} - (p^{\mathrm{s}} + p^{\mathrm{tc}}) + \beta \mathbb{E}[J'(\theta')|J,\theta], \quad (12)$$

the participation constraints $J'(\theta') \geq 0$, and the feasibility requirement that $p^{\mathrm{s}} + p^{\mathrm{tc}} \leq x^{\eta}$.

The outcome of this problem is a policy function $\{x(J,\theta), p^{\mathrm{s}}(J,\theta), p^{\mathrm{tc}}(J,\theta)\}$ and a law of motion for future promises for each possible state θ', $J'(\theta'|J,\theta)$. These objects fully characterize the behavior of the economy with trade credit in this example.

Before moving on to study the properties of this economy, it is useful to derive the first-order conditions. Let μ, ι, and κ be the Lagrange multipliers associated, respectively, with constraints (9), (10), and (11), and let λ be the multiplier associated with the promise-keeping constraint (12). After some rearrangement, we obtain[22]

$$\begin{aligned} \{[\delta + (1-\theta)(1-\delta)]\mu + \lambda\}\,\eta x^{\eta-1} &= W(1+\iota), \quad (13) \\ 1+\iota &= \mu + \lambda, \quad (14) \\ 1 + (1-\theta)\iota &= \kappa + \lambda, \quad (15) \\ \lambda'(\theta') &= \lambda + \kappa. \quad (16) \end{aligned}$$

Equation (13) is the optimality condition for x. Relative to the spot economy, increasing x has the additional benefit of relaxing the promise-keeping constraint, an effect represented by the Lagrange multiplier λ on the left hand side of the equation. It also has an additional cost when the supplier debt limit binds, as wages need to be paid in the morning and those resources have a shadow cost when the suppliers' are constrained, an effect that is represented by the Lagrange multiplier ι on the right hand side of the equation.

Equations (14) and (15) are the optimality conditions with respect to p^{s} and p^{tc}. A marginal increase in p^{s} raises the profits of the supplier by one unit and relaxes their

[22] In what follows, we assume that the participation constraint and the feasibility requirements do not bind in equilibrium. In the steady state, this will be true as long as $\theta > 0$.

borrowing constraint with the banks (when it binds), which explains why the marginal benefit equals $1 + \iota$. The marginal cost of increasing p^s is a tightening of the borrowing constraint of final good firms and of the promise-keeping constraint, $\mu + \lambda$. The trade-off for p^{tc} is very similar. The key difference between p^s and p^{tc} is that the latter does not relax the borrowing constraint of the supplier as much because banks advance at most a fraction $(1 - \theta)$ of the supplier's accounts receivable. When the borrowing constraint of the monopolist does not bind ($\iota = 0$), we have that $\mu = \kappa$ and the supplier is indifferent between being paid spot or in credit. When the borrowing constraint of the supplier binds ($\iota > 0$), giving trade credit to customers is costly for the supplier relative to being paid spot in the morning. *Ceteris paribus*, a tightening of the supplier's borrowing constraint reduces the incentives to extend trade credit to its customers.

Equation (16) combines the first-order condition of the problem with respect to $J'(\theta')$ and the envelope condition. This equation describes a law of motion for the Lagrange multiplier on the promise-keeping constraint, λ. We can see that the multiplier grows over time whenever the trade credit constraint binds, $\kappa > 0$. This is quite intuitive: the supplier has an incentive to increase future rents for his customers when the trade credit constraint binds because this allows them to extend more trade credit today. As the rents of final good firms grow over time, so does the Lagrange multiplier on the promise-keeping constraint.

3.1 The credit multiplier

We start by considering the case in which θ_t is deterministic and equal to $\tilde{\theta}$ for all t. The next proposition characterizes the limit of the optimal contract as $t \to \infty$.

Proposition 1. *Let $\bar{\theta}$ be such that $\delta + (1-\delta)(1-\bar{\theta})(1+\beta\bar{\theta}) = \eta$. If $\tilde{\theta} \leq \bar{\theta}$, the borrowing constraint of the monopolist does not bind in steady state and the optimal contract converges to*

$$\begin{aligned} x = \left[\frac{\eta}{W}\right]^{\frac{1}{1-\eta}}, \qquad p^s = [\delta + (1-\delta)(1-\theta)]x^{\eta} \\ p^{tc} = \beta(1-\delta)\tilde{\theta}x^{\eta}, \qquad J' = (1-\delta)\tilde{\theta}x^{\eta}. \end{aligned} \tag{17}$$

If $\tilde{\theta} > \bar{\theta}$, the borrowing constraint of the monopolist binds in steady state and the optimal contract converges to

$$\begin{aligned} x = \left[\frac{\delta + (1-\delta)(1-\tilde{\theta})(1+\beta\tilde{\theta})}{W}\right]^{\frac{1}{1-\eta}}, \qquad p^s = [\delta + (1-\delta)(1-\theta)]x^{\eta} \\ p^{tc} = \beta(1-\delta)\tilde{\theta}x^{\eta}, \qquad J' = (1-\delta)\tilde{\theta}x^{\eta}. \end{aligned} \tag{18}$$

Proposition 1 identifies two regions of the parameter space. When $\tilde{\theta}$ is smaller than the threshold $\bar{\theta}$, there are no output distortions in steady state—as x is such that the marginal product of labor equals the wage. When $\tilde{\theta}$ is greater than $\bar{\theta}$, output is distorted down relative to the efficient level, with the size of the distortion increasing in $\tilde{\theta}$.

Figure 1 presents a numerical illustration. The solid line plots the steady state level of output, total bank credit to all firms in the economy, and trade credit in the benchmark economy as a function of $\tilde{\theta}$. The dashed line in the middle panel represents bank credit to final good firms. We can see that in the first region $\tilde{\theta}$ affects the financing of the supply chain, but not the overall output produced. Specifically, the higher is $\tilde{\theta}$, the less final good firms borrow from banks and the more they borrow from their supplier to finance their input payments in the morning.

As we increase $\tilde{\theta}$, the supplier provides more trade credit to the costumers and starts borrowing from banks to pay for the wages of workers in the morning. Eventually, the supplier borrows so much that their borrowing constraint with the banks (10) binds. In the picture, this happens when $\tilde{\theta} = \bar{\theta}$. From that point on, the economy falls in the second region: higher levels of $\tilde{\theta}$ are associated with lower bank *and* trade credit for final good firms, and with lower output.

The figure also compares the steady state of the benchmark economy to that of the spot economy (line with circles). The economy with trade credit always features a higher level of output relative to the spot economy, with the distance between the two economies increasing when $\tilde{\theta} < \bar{\theta}$ and decreasing otherwise. Thus, trade credit relationships alleviate the economic costs of credit market frictions, especially when the supplier has untapped borrowing capacity.

Why is the trade credit economy producing more output in steady state than the spot economy? It is the product of two forces.

First, the supplier in the trade credit economy obtains a larger share of final good firms' revenues relative to what they get in the spot economy.[23] The fact that the supplier captures a larger shares of the revenues than in the spot economy compresses the wedge between the marginal product of labor and the wage, an effect that can be seen by comparing the left hand side of the first order condition for x across the two economies, equations (7) and (13). This simply means that in the trade credit economy the upstream firm has more incentives to sustain the production of the supply chain. In addition, the trade credit economy can channel more resources to the payments of labor costs in the morning than

[23] In the spot economy, payments to the supplier only occur in the morning, so they cannot exceed a fraction $\delta + (1-\delta)(1-\tilde{\theta})$ of total revenues. In the trade credit economy, payments to the supplier also occur in the afternoon, so they are larger.

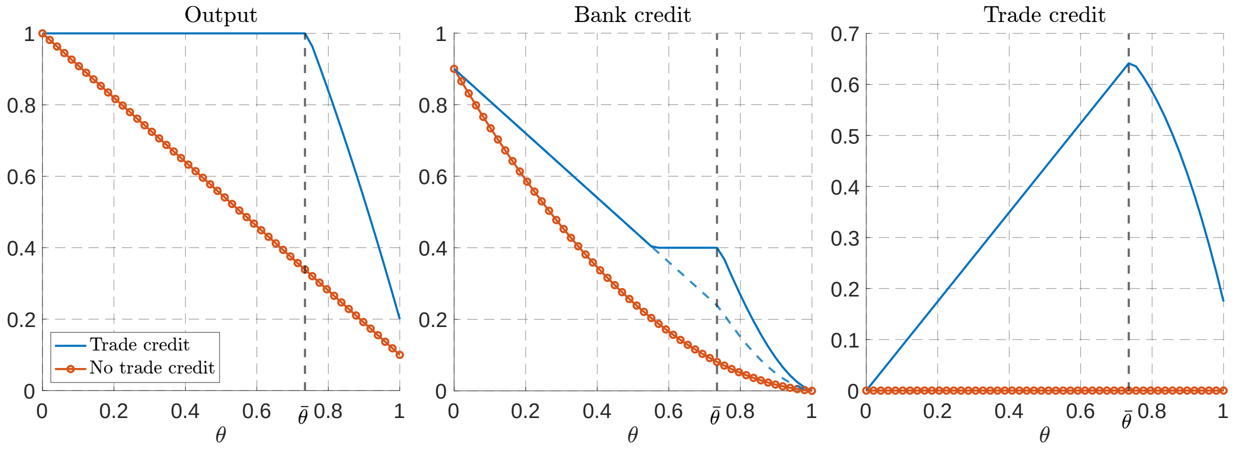


Figure 1: The deterministic steady state

Note: For the numerical illustration, we set $\beta = 0.97$, $\eta = 0.5$, $\chi = 0.5$, and $\delta = 0.1$. The solid blue line reports the steady state level of output, total bank credit, and trade credit to final good firms in the benchmark economy for different values of $\tilde{\theta}$. The red line with circles reports the same values for the spot economy. The dashed blue line in the middle panel reports bank credit to final good firms.

the spot economy, and it can ultimately support a higher level of production.

To better understand this last point, it is useful to first consider the spot economy. There, bank credit flows to final good firms and from them to the supplier. The supplier, then, uses these morning funds for two purposes: paying for their production costs (the workers' wages) and remunerating his rents.[24] Effectively, the payments of supplier's rents in the morning restricts how much resources can be directed to the payments of labor.

Second, in the economy with trade credit the supplier can direct more funds to cover their production costs in the morning. This happens both because the economy as whole can sustain a larger amount of bank credit, and because a larger share of bank credit can be directed towards labor. The supplier in the trade credit economy can borrow from banks up to a fraction $(1 - \tilde{\theta})$ of their accounts receivable, while in the spot economy this doesn't happen. The process of discounting trade credit bills with banks allows the economy to have more bank credit than the spot economy: in the latter, bank credit is at most $(1 - \delta)(1 - \tilde{\theta})x^{\eta}$, while it is at most $(1 + \beta\tilde{\theta})(1 - \delta)(1 - \tilde{\theta})x^{\eta}$ in the economy with trade credit. In addition, the supplier in the trade credit economy can shift part of the remuneration of their rents to the afternoon—reducing the crowding out effects.

Discussion. Summarizing this section, we note that trade credit has two main effects in the model: it increases the overall loan *quantity* that banks transfer to firms, and it

[24] Indeed, from equation (7), we can see that a fraction $[\delta + (1 - \delta)(1 - \tilde{\theta})]\eta$ of the revenues of final good firms is directed toward labor payments, while a fraction $[\delta + (1 - \delta)(1 - \tilde{\theta})](1 - \eta)$ goes toward the supplier's rents.

improves its *allocation*—as a larger share of it is directed toward the payments of productive inputs. We now further discuss the logic behind the credit multiplier and link it to existing empirical evidence.

The trade credit economy supports more bank credit than the spot economy because the former has a larger pool of borrowers. In the spot economy, there is no reason for suppliers to borrow from banks, as they receive all of their payments in the morning. In the trade credit economy, instead, suppliers receive some of their revenues in the afternoon while all their costs still occur in the morning; this increases their demand for bank credit. So, effectively, in our model, banks provide the working capital necessary for suppliers to produce, while the latter offer credit to their costumers in the form of delayed payments. This process of invoice discounting, which is at the heart of our mechanism, is quite common in practice. In Italy, for example, advances on trade credit bills by banks and other financial institutions amounted to 118 billion euros in the last quarter of 2019, which represents approximately 40% of all short-term credit provided to non-financial corporations.[25] Similarly, Caglio, Darst, and Kalemli-Ozcan (2021) finds that roughly one-third of all loans to US firms are backed by accounts receivables and inventories.

Even though it is ultimately financed by banks, our model shows that trade credit is not merely a "veil"—in the sense that these inter-firm financial linkages are a necessary ingredient to increase the overall quantity of bank credit flowing to firms. To understand why, consider an equilibrium of the spot economy in which the final good firms are financially constrained. In such equilibrium, banks are not willing to lend an extra dollar toward the production of the final good because they are afraid that the final good firm would default. Why are they willing to lend this extra dollar in an economy with trade credit? Because they will give it to the supplier, who is not levered and therefore has no incentives to default; and the supplier will lend it to the costumer in the form of delayed payment, because they know that the costumer would not default out of fear of losing access to the intermediate input going forward. So, trade credit introduces an additional cost of default for the borrower, which is necessary to sustain more debt in equilibrium. In this argument, it is important that the supplier keeps the ownership of the trade credit bill when discounting it with the bank. While we are not aware of systematic studies on this topic, it is remarkable that the vast majority of invoice discounting operated by Italian banks is "*salvo buon fine*"—with the suppliers keeping the credit risk implicit in the trade credit bill.

[25] Advances on trade credit bills are obtained from aggregated data of the Italian credit registry, accessed from the *Base Dati Statistica* of the Bank of Italy. We take the loan category "Rischi autoliquidanti: utilizzato" (Table TRI30101) and we subtract "Prestiti cessione stipendio" (Table TFR20281). The resulting figure includes factoring services to non-financial corporations and other forms of advances of trade credit bills offered by banks and other financial institutions.

In addition to increasing the quantity of credit in the economy, the presence of trade credit improves its allocation—in the sense that a larger fraction of bank credit is directed toward paying for the wages of workers rather than the rents of suppliers. This mechanism is closely linked to what Garcia-Marin, Justel, and Schmidt-Eisenlohr (2019) in previous work have referred to as the "financing cost advantage of trade credit", see their paper for empirical evidence on this mechanism.

3.2 Trade credit and the response to financial shocks

We now turn to study how the two economies respond to an increase in θ. This shock tightens the supply of credit by banks, so the comparison is informative about whether the presence of trade credit dampens or amplifies the economic effects of financial shocks. To this purpose, we assume that θ can take two values: $\{\tilde{\theta}, \tilde{\theta} + \varepsilon\}$ with $\epsilon > 0$ but small and transition matrix $p(\theta'|\theta)$. We then study how output responds following a switch from the low to the high-θ state.

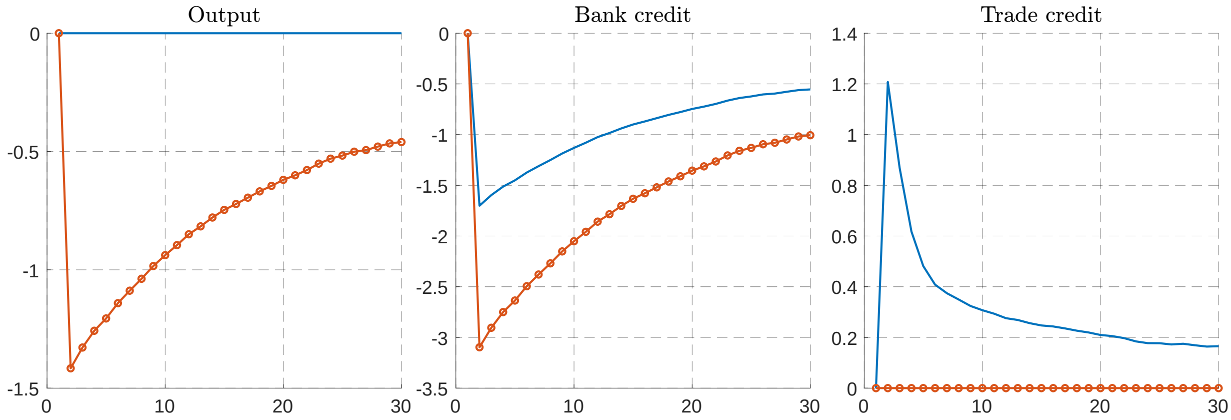
The output effect of this shock in the spot economy can easily be studied using equation (8):[26] in response to an increase in θ, credit supply shrinks, final good firms demand less intermediate inputs, and the economy produces less output. This effect is stronger the more levered the supply chain is—the larger is α^{spot}. In the benchmark economy, the output effects of the same shocks are more subtle, and they depend on whether or not the supplier is financially constrained.

Proposition 2. *Suppose that $\tilde{\theta} < \bar{\theta}$. Let $\theta = \tilde{\theta}$ for a sufficiently long time, and consider a switch to $\theta = \tilde{\theta} + \varepsilon$ where ε is small enough so that $\tilde{\theta} + \varepsilon \leq \bar{\theta}$. Then, output does not change in response to the shock.*

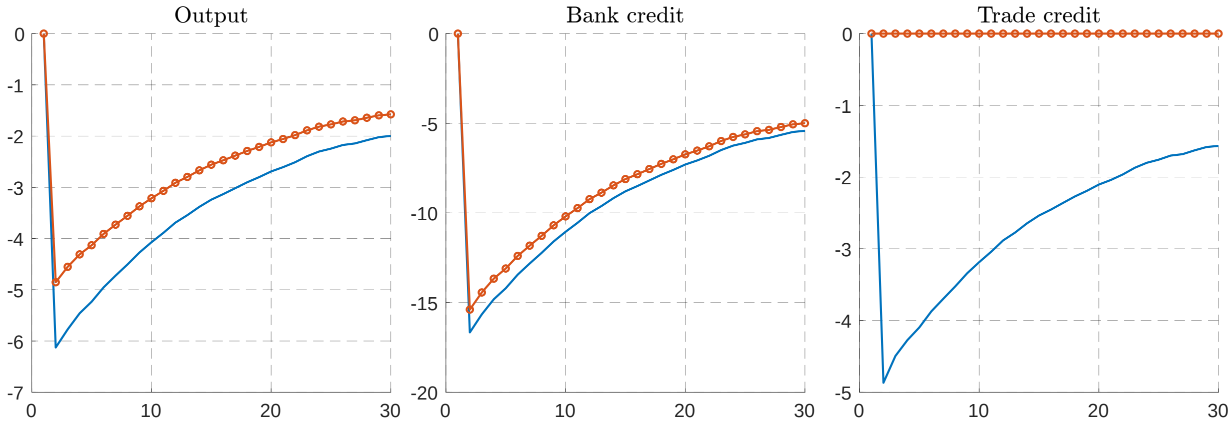
When $\tilde{\theta} < \bar{\theta}$ the financial constraint of the supplier does not bind in a deterministic steady state. In this scenario, a small financial shock has no effects on the operations of the supply chain: following the financial shock, the supplier extends more trade credit to the customers and this compensates for the fall in bank credit they experience. The solid lines in panel (a) of Figure 2 provide a numerical illustration of this case, and they can be compared to the behavior of the spot economy (line with circles). As we can see, the presence of trade credit dampens the output effect of financial shocks.

When $\tilde{\theta} > \bar{\theta}$, the borrowing constraint of the supplier binds in the steady state. In this region, the supplier will not be able to provide liquidity to its customers, so a financial

[26] We can use the comparative statics result of the spot economy to understand the behavior of the economy in response to a financial shock, as the spot economy does not feature internal dynamics.



(a) Dampening of financial shocks



(b) Amplification of financial shocks

Figure 2: Impulse response function to a financial shock

Note: For the numerical illustration, we set $\beta = 0.97$, $\eta = 0.5$, $\chi = 0.5$, $\delta = 0.1$, $\varepsilon = 0.01$, $p(\tilde{\theta}|\tilde{\theta}) = 0.99$, and $p(\tilde{\theta} + \varepsilon|\tilde{\theta} + \varepsilon) = 0.95$. For the top panel, we set $\tilde{\theta} = 0.4$, while for the bottom panel we set $\tilde{\theta} = 0.9$. The solid blue line reports the response of output, total bank credit, and trade credit to final good firms in log-deviations (pts) from their steady state values in the benchmark economy. The red line with circles reports the same information for the spot economy. Impulse response functions are computed by simulations following the methodology in Koop, Pesaran, and Potter (1996).

shock will have effects on output even in the economy with trade credit. Unfortunately, we cannot provide a sharp analytic characterization of the impulse response function in this case, and whether the overall output effects of the shocks are larger or smaller than those of the spot economy will typically depend on model parameters. Panel (b) of Figure 2 provides a numerical illustration of a case in which trade credit amplifies the negative effects of financial shocks. We can see that bank credit and output fall *more* in response to the financial shock in the economy with trade credit than in the spot economy.

To understand why trade credit can amplify the effects of financial shocks on output when $\tilde{\theta} > \bar{\theta}$, consider a perturbation of the steady state level of output with respect to $\tilde{\theta}$ in this region of the parameter space. Using the expression for x in Proposition 1 and rearranging terms, we obtain

$$\varepsilon^{\text{tc}}_{x,\theta} = -\frac{1}{1-\eta}\frac{\tilde{\theta}}{1-\tilde{\theta}}\alpha^{tc} - \frac{1}{(1-\eta)}\frac{(1-\tilde{\theta})\beta(1-\delta)\tilde{\theta}}{\delta+(1-\delta)(1-\tilde{\theta})(1+\beta\tilde{\theta})}, \tag{19}$$

where $\alpha^{\text{tc}} \equiv \frac{(1-\delta)(1-\tilde{\theta})(1+\beta\tilde{\theta})}{\delta+(1-\delta)(1-\tilde{\theta})(1+\beta\tilde{\theta})}$ is the leverage of the supply chain in the economy with trade credit.

The first term on the right hand side of equation (19) captures the same mechanism present in the spot economy and described in equation (8), whereby a fall in credit supply has larger effects the higher the leverage of the supply chain. Importantly, and because of the credit multiplier effect discussed in the previous subsection, the leverage of the supply chain in the trade credit economy is larger than that of the spot economy when the borrowing constraint of the supplier binds, $\alpha^{\text{tc}} > \alpha^{\text{spot}}$. So, this force contributes to making output in the trade credit economy more sensitive to financial shocks. The second term in the expression captures a countervailing force. Holding x constant, an increase in $\tilde{\theta}$ increases the surplus that customers get in equilibrium, and this allows the supplier to offer more trade credit. This force—which is absent in the spot economy—mitigates the effect of the financial shock on output. Which force dominates depends on model parameters. In the steady state, one can show that $\left|\varepsilon^{\text{tc}}_{x,\theta}\right| > \left|\varepsilon^{\text{spot}}_{x,\theta}\right|$ if and only if $\tilde{\theta} > \frac{1}{1+\sqrt{\delta}}$. That is, the trade credit economy has a larger sensitivity of output to $\tilde{\theta}$ only when financial conditions are severe enough.

4 Sectoral heterogeneity and computations

The special case studied in the previous section abstracts from two main ingredients that are instead present in our model. First, the economy of Section 2 has many supply chains with heterogeneous characteristics, each with potentially many suppliers. Second, the fully fledged model has general equilibrium forces.

Despite these differences, most of the results we discussed in the previous section extend to the full model. Indeed, the decision problem of one supplier operating in sector i is mathematically very similar to the monopolist case studied in Section 3, and it can be written recursively in terms of promised surplus for the final good firms $\tilde{J}_i$ and the aggregate financial shock θ. While we leave the analysis of that problem to Appendix A.1, we now discuss some properties of the optimal contract, starting from the deterministic steady state.

4.1 The deterministic steady state

The following proposition characterizes the deterministic steady state of the optimal trade credit contract offered by a supplier in sector i.

Proposition 3. *Fix (W, C) and let*

$$rev_i(x) = C^{\frac{1}{\gamma}} k^{\frac{\gamma-1}{\gamma}(1-\eta_i)} (x)^{\eta_i \frac{\gamma-1}{\gamma}} N_i^{\eta_i \frac{\sigma}{\sigma-1} \frac{\gamma-1}{\gamma}}$$

be the revenues of final good firms operating in production line i as a function of x. Let

$$x_i^{unc} = \left[\frac{k^{(1-\eta_i)\frac{\gamma-1}{\gamma}} \eta_i \frac{\gamma-1}{\gamma} C^{\frac{1}{\gamma}}}{W N_i^{1-\eta_i \frac{\sigma}{\sigma-1} \frac{\gamma-1}{\gamma}}} \right]^{\frac{\gamma}{\eta_i + (1-\eta_i)\gamma}},$$

and define x_i^{con} implicitly from the expression

$$\begin{aligned} W x_i^{con} &= \Bigg\{ [1 - \tilde{\theta}(1 - \delta + \delta \pi_i)] + (1 - \tilde{\theta}) \beta N_i \Bigg[N_i \Bigg(1 - \left(\frac{N_i - 1}{N_i} \right)^{\eta_i \frac{\sigma}{\sigma-1} \frac{\gamma}{\gamma-1}} \Bigg) \\ &- \frac{1}{N_i} [1 - \tilde{\theta}(1 - \delta + \delta \pi_i)] \Bigg] \Bigg\} \frac{rev_i \left(x_i^{con} \right)}{N_i}. \end{aligned} \tag{20}$$

There exist two thresholds, $\underline{\theta}_i$ and $\overline{\theta}_i$, with $\underline{\theta}_i$ defined as

$$\underline{\theta}_i = \left\{ 1 - N_i \left[1 - \left(\frac{N_i - 1}{N_i} \right)^{\eta_i \frac{\sigma}{\sigma-1} \frac{\gamma-1}{\gamma}} \right] \right\} \frac{1}{(1-\delta) + \delta \pi_i} \tag{21}$$

and $\overline{\theta}_i$ being the θ guaranteeing that equation (20) holds with equality when $x_i^{con} = x_i^{unc}$, such that

1. *If $\tilde{\theta} \leq \underline{\theta}_i$, the optimal contract offered by the suppliers in production line i converges to*

$$\begin{aligned} x_i &= x_i^{unc} \\ p_i^s &= N_i \left[1 - \left(\frac{N_i - 1}{N_i} \right)^{\eta_i \frac{\sigma}{\sigma-1} \frac{\gamma-1}{\gamma}} \right] rev_i(x_i^{unc}) \\ p_i^{tc} &= 0 \end{aligned}$$

2. *If $\tilde{\theta} \in (\underline{\theta}_i, \overline{\theta}_i]$, the optimal contract offered by the suppliers in production line i converges to*

$$\begin{aligned} x_i &= x_i^{unc} \\ p_i^s &= \{1-\theta(1-\delta+\delta\pi_i)\}\, rev_i(x_i^{unc}) \\ p_i^{tc} &= \beta N_i \left\{ \left[1 - \left(\frac{N_i - 1}{N_i} \right)^{\eta_i \frac{\sigma}{\sigma-1} \frac{\gamma-1}{\gamma}} \right] - \frac{1}{N_i} \left[1 - \theta(1-\delta+\delta\pi_i) \right] \right\} rev_i(x_i^{unc}) \end{aligned}$$

3. *If $\tilde{\theta} > \overline{\theta}_i$, the optimal contract offered by the suppliers in production line i converges to*

$$\begin{aligned} x_i &= x_i^{con} \\ p_i^s &= \{1-\theta(1-\delta+\delta\pi_i)\}\, rev_i(x_i^{con}) \\ p_i^{tc} &= \beta N_i \left\{ \left[1 - \left(\frac{N_i - 1}{N_i} \right)^{\eta_i \frac{\sigma}{\sigma-1} \frac{\gamma}{\gamma-1}} \right] - \frac{1}{N_i} \left[1 - \theta(1-\delta+\delta\pi_i) \right] \right\} rev_i(x_i^{con}) \end{aligned}$$

The structure of the contract is similar to that of the special case of Section 3, with trade credit being used by the supplier only when the financial constraint of the costumer binds.[27] Differently from the special case, there are now three regions to consider, depending on the value of $\tilde{\theta}$.

In the first region, which occurs when $\tilde{\theta}$ is below the cutoff $\underline{\theta}_i$, the supply chain produces at the financially unconstrained level of output x_i^{unc} and its operations are entirely financed by bank credit, so $p_i^{\text{tc}} = 0$. This possibility was also present in the special case of Section 3, but it occurred only when $\tilde{\theta} = 0$. In that economy, an unconstrained monopolist wants to extract all the rents from the costumer. Because that requires the final good firm to transfer all the revenues to the monopolist, the borrowing constraint of the final good firm necessarily binds unless $\tilde{\theta} = 0$. In the full model, instead, an unconstrained monopolist cannot extract all the rents from the costumer when $N_i > 1$ because of competitive forces. Therefore, there is range of values of $\tilde{\theta}$ in which the borrowing constraints of the final good firms do not bind and the contract supports an efficient level of output.

In the second region, which occurs when $\tilde{\theta} \in (\underline{\theta}_i, \overline{\theta}_i]$, the supply chain still produces at the financially unconstrained level of output, but the financial constraints of the final good producers bind. Therefore, suppliers need to extend trade credit to support the allocation. This region corresponds to region 1 in Proposition 1.

[27]Importantly, this implies a "pecking-order" between bank and trade credit in the model. Trade credit is, in fact, costly for suppliers as they need to distort up future rents of its costumer in order to incentivize repayment by the costumer. So, the supplier will use it as a last resort only when the costumer is financially constrained. This is view is consistent with the corporate finance literature, see Petersen and Rajan (1997).

Finally, the third region occurs when $\tilde{\theta} > \overline{\theta}_i$. In this space, the suppliers in sector i are financially constrained in steady state, and output is distorted downward relative to the efficient level—as it was in the case in region 2 in Proposition 1.

Comparative statics. Proposition 3 is also useful to study how production and financing decisions of the supply chain varies with industry characteristics, which in our model are represented by the parameters $\{\eta_i, \pi_i, N_i\}$.

Lemma 1. *Suppose that the economy is in steady state, and consider a supply chain i. If $\theta > \underline{\theta}_i$, we have the following comparative static results:*

$$\frac{\partial \left(p_i^{tc}/rev_i\right)}{\partial \eta_i} > 0, \qquad \frac{\partial \left(p_i^{tc}/rev_i\right)}{\partial \pi_i} > 0, \qquad \frac{\partial \left(p_i^{tc}/rev_i\right)}{\partial N_i} < 0.$$

Lemma 1 tells us that supply chains with high intermediate inputs share (high η_i), a high share of accounts receivable over sales (high π_i), and that source intermediate goods from concentrated markets (low N_i) will have more trade credit as a fraction of their revenues. These results are quite intuitive, as we discuss next.

A high η_i and high π_i mean that final good firms in sector i have a large need for working capital in the morning because they need to purchase more intermediate inputs to produce their output but receive little cashflow in the morning as they sell what they produce in credit to households. Therefore, their demand for funds in the morning will be high and, ceteris paribus, the supply chain will require more trade credit to operate.

A low N_i means that there are few suppliers providing intermediate goods in sector i. Because of that, losing a relationship to one of them will be quite costly for the final good firms. As a result, the final good firms in low N_i sectors will have less incentives to default on their suppliers, which allows the supply chain to sustain more trade credit in equilibrium.

In Section 5 we will use Italian data to test these cross-sectional predictions of the model.

4.2 Dynamic adjustment to financial shocks

The analysis of how the economy adjusts to financial shocks in the full model is somewhat different from the special case considered in Section 3 because financial shocks now trigger general equilibrium effects that were absent in the previous analysis. These forces are also present in the spot economy, and they can be studied analytically in this case. After some manipulations of the optimality condition for labor, we can derive the elasticity of x_i in sector i to a marginal increase in θ in the spot economy:

$$\varepsilon_{x_i,\theta} = \frac{-\theta\left(1-\delta+\delta\pi_i\right)}{\left[1-\theta\left(1-\delta+\delta\pi_i\right)\right]\left(1-\eta_i\right)} + \frac{1}{1-\eta_i}\left(\frac{1}{\gamma}\varepsilon_{C,\theta} - \varepsilon_{W,\theta}\right).$$

The first part of the right hand side of this expression is identical to (8), after factoring in the fact that we set $\pi_i = 0$ in the special case. The second part of the expression isolates the general equilibrium forces. When θ increases, it reduces the demand for all the other goods in the economy, $\varepsilon_{C,\theta} < 0$: to the extent that varieties are not perfectly substitutable ($\gamma < \infty$), the demand for variety i will fall as a result. This "aggregate demand" channel amplifies the impact of the shock on the output produced by sector i, and it is stronger the smaller is γ. In addition, an increase in θ lowers labor demand and depresses wages, $\varepsilon_{W,\theta} < 0$. This reduces the cost of production and dampens the effect of the shock on output.

These two general equilibrium forces are at play in the benchmark model too, and they will contribute to shaping the response of the economy to financial shocks. In the next section we will solve numerically a calibrated version of the fully fledged economy and use it to assess whether trade credit dampened or amplified the financial shocks associated with the Great Recession in our application.

Solving the model is not a simple task. As we show in Appendix A.1, one can set up the supplier's problem recursively in the spirit of Thomas and Worrall (1988), where the state variables in the economy include not only the exogenous financial state θ, but also the surplus each supplier promised their customers in the previous period. Since sectors are heterogeneous but suppliers in each sector are identical, this amounts to one additional state variable per sector $\{\tilde{J}\}_i$.

When suppliers choose the optimal trade credit contract, they need to take into account not only the current wage W and aggregate demand C, but also their values in the following period. This is because, in every period, suppliers have to choose what surplus they promise to deliver to their customers for each realization of the aggregate financial shock in the following period. In theory, there is a mapping between the state of the economy $\left\{\theta, \{\tilde{J}\}_i\right\}$ and the current wage and aggregate consumption. In practice, solving for that mapping, which depends on 59 different state variables, is not computationally feasible. To overcome the challenge, we follow the logic of Krusell and Smith (1998) and approximate the evolution of W and C using an autoregressive law of motion.

We provide a detailed description of the numerical algorithm in Appendix A.4. Here, let us briefly sketch the algorithm. Our approach relies on policy function iteration combined with an approximate law-of-motion for the level of the aggregate wage and for the level of aggregate consumption. We conjecture that these two variables $\{W, C\}$ follow an AR(1) process in logs, whose coefficients depend on the current state of the aggregate financial shock, θ. So that the law of motion for aggregate consumption, for example, is given by

$$\ln C_t = (1 - \rho_c(\theta_t))\mu_c(\theta_t) + \rho_c(\theta_t) \ln C_{t-1}. \tag{22}$$

The law of motion for the aggregate wage follows a similar specification. Our calibrated model features two values of θ, so that these laws of motion are characterized by 8 coefficients. Given these laws of motion, we can solve the problem of each sector independently, where there are only four state variables for each sector: (i) the aggregate financial shock θ, (ii) the promised surplus in the sector $\tilde{J}_i$, (iii) the aggregate wage W, and (iv) the aggregate level of demand C. We solve each sector using policy function iteration. Once we find the policy functions, we can simulate the economy for a large number of periods and obtain realized values of $\{\theta_t, W_t, C_t\}$. We then estimate the coefficients of the AR(1) processes via an OLS regression. With the new estimated coefficients we can repeat the process—solve for the policy functions in each sector, simulate the model, and obtain estimates of the AR(1) coefficients.[28] We repeat the process until the eight AR(1) coefficients converge.[29]

5 Quantitative analysis

We now move on to a quantitative analysis of the macroeconomic implications of trade credit. Section 5.1 describes the data, and Section 5.2 presents the calibration and discusses the in-sample and out-of-sample properties of the model. Section 5.3 presents the main counterfactuals—one aimed at assessing the size of the credit multiplier and one studying the role of trade credit during the Great Recession. Section 5.4 concludes with a discussion of some policy implications.

5.1 Data

We use Italian firm-level data between 2007 and 2015 from the historical ORBIS dataset compiled by Bureau van Dijk. We study a balanced panel of non-financial corporations. Appendix A.3 lays out the cleaning procedure. For each firm in our panel, we observe at an annual frequency operating revenues, sales, short-term bank loans, a measure of expenditures on intermediate inputs, the sector in which the firm operates, as well as accounts payable and receivable. Accounts payable is the amount that the firm owes for

[28] In the algorithm, when we update the AR(1) coefficients, we take a convex combination of the previous guess and the resulting implied AR(1) estimates.

[29] We note that the resulting AR(1) processes we estimate in the final iteration of our calibration are highly accurate. The $\mathbb{R}^2$ of the two laws of motion are higher than 0.99999.

goods it already received, while accounts receivable is the amount that the firm needs to receive for goods that it has already sold.[30]

We map the data to our model as follows. Using the classification from the Italian input-output tables, we partition the firms in our panel into 58 different sectors. We then average the firm-level balance sheet items for firms within each sector, obtaining the sectoral-level data. Each sector corresponds to a different supply chain in our model, and the balance sheet data are mapped to the corresponding item for the final good firms.

Given the above assumption, we map the share of accounts payable over revenues for sector i to $p^{\text{tc}}_{i,t}/rev_{i,t}$. In addition, the sectoral balance sheet items provide information on key parameters of the model. The share of expenditures on intermediate inputs (materials and services) over operating revenues—$(p^{\text{s}}_{i,t} + p^{\text{tc}}_{i,t})/rev_{i,t}$ in the model—is informative about η_i.[31] The share of the accounts receivable of final good firms over their sales, $\delta\pi_i$ in the model, is informative about π_i. The ratio of short-term bank loans to sales provides information about θ_t, as it is equal to $[1 - \theta_t(1 - \delta + \delta(1 - \pi_i))]$ in the model when the financial constraint of final good firms in sector i binds.

We also use the sectoral data to construct an empirical counterpart to N_i, the degree of concentration of suppliers' in sector i. To do so, we exploit the fact that in the symmetric equilibrium suppliers have the same sales shares, so $1/N_i$ corresponds to the Herfindahl-Hirschman index (HHI) of the suppliers' market for sector i. To construct this object in the data, we use ORBIS and obtain the HHI for each of the 58 sectors in our dataset. We then use Italy's input-output tables to construct a weighted average HHI of a sector's suppliers. The weight of sector j in this calculation is equal to the share of inputs provided to sector i by sector j.[32]

Table 1 provides descriptive statistics on the sector-level variables. The ratio of accounts payable to sales is 23% on average, which is 40% higher than the average ratio of short-term loans to sales. The ratio of accounts receivable to sales is 29% on average. There is large variation across sectors in all three of these ratios. The average ratio of expenditures on intermediate inputs to sales is 67%. Finally, the average $\text{HHI}^{\text{supplier}}$ is 0.05. This value corresponds to a market operated by 20 identical suppliers.

Before moving on to the calibration, we use the dataset to test some of the key predictions of the model. We start by studying the cross-sectoral relations described in Lemma 1. There,

[30] The Italian ORBIS dataset does not contain an explicit variable with expenditures on intermediate inputs. To construct this variable, we subtract earning before income and tax (EBIT), the wage bill, and depreciation from operating revenues.

[31] From the expressions in Proposition 3, we can see that in a deterministic steady state, this ratio is equal to $\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}$ when N_i is large.

[32] We construct this index using 2010 data, as it is the only year in our sample where we have Italian data on input-output relationships at a fine sectoral level.

Table 1: Sector-level descriptive statistics

	Mean	St. deviation	Min	Max
Accounts payable/sales	0.23	0.05	0.10	0.39
Accounts receivable/sales	0.29	0.08	0.07	0.59
Short-term bank loans/sales	0.16	0.09	0.07	0.62
Intermediate inputs /sales	0.67	0.08	0.41	0.84
$\text{HHI}^{\text{supplier}}$	0.05	0.02	0.02	0.14

Notes: This table presents descriptive statistics on sector-level variables of interest. The sample includes the 58 sectors in Italy's input-output table in 2007. The value of supplier's HHI is computed in 2010 because of data limitations.

we showed that in a deterministic steady state, final good firms in sectors with high η_i, π_i and a low N_i obtain less trade credit from their suppliers. We can evaluate these predictions by performing simple linear regressions, with results reported in Table 2.

The dependent variable in all specifications is the ratio of accounts payable to sales for each sector at time t. Specification (1) shows that accounts receivable are positively associated with accounts payable, with a quite strong relationship (an adjusted R^2 of 0.33). Specification (2) shows that there is also a significant positive relationship between expenditures on intermediate inputs—a proxy for η_i—and accounts payable, as our model would predict. Specification (3) shows a significant and positive relationship between suppliers' HHI and accounts payable. That is, firms that receive intermediate inputs from more concentrated markets tend to pay more in credit. This result is consistent with the findings in Garcia-Marin, Justel, and Schmidt-Eisenlohr (2019) of a positive relationship between a suppliers' markups and the amount of trade credit they offer to costumer using Chilean transaction-level data. It is also consistent with the predictions of our theory, whereby firms are more willing to pay their trade credit debt if the supplier has more market power. Specification (4) reports the regression with all three control variables. All coefficients are significant, and their signs are consistent with Lemma 1. These three factors jointly account for 48% of the variation in accounts payable across sectors.

A second important prediction of the model is that firms' production choices are more sensitive to financial shocks the tighter the financial constraints of their suppliers. To test this prediction, we follow a diff-in-diff approach: we use the calibrated model (see the next subsection) to sort sectors according to the borrowing capacity of their suppliers in 2007 and check whether sectors where suppliers are more financially constrained experienced a deeper fall in trade credit and output during the Great Recession, which we interpret as a financial shock.

We use the results in Proposition 3 along with data to compute the threshold level of θ

Table 2: Cross-sectional sector-level regressions

	Dep. variable: Accounts payable/sales			
	(1)	(2)	(3)	(4)
Accounts receivable/sales	0.297***			0.326***
	(0.029)			(0.027)
Intermediate inputs /sales		0.154***		0.239***
		(0.028)		(0.020)
$HHI^{supplier}$			0.655***	0.303***
			(0.111)	(0.085)
Adj. R^2	0.329	0.092	0.116	0.477
Obs.	522	522	522	522

Notes: This table presents the sector-level regression results. Consistent with the model's prediction, we find that a high ratio between accounts payable and sales is associated with: (i) a high ratio of accounts receivable to sales, (ii) a high ratio of intermediate inputs to sales, and (iii) a high degree of HHI among the sector's suppliers. All regressions include year fixed effects. Robust standard errors are in parentheses. *** - significant at the 1% level.

at which the suppliers in sector i are financially constrained, $\overline{\theta}_i$. According to our model, sectors with a relatively low value of $\overline{\theta}_i$ are more likely to have financially constrained suppliers at any given point in time than sectors with high $\overline{\theta}_i$. Our approach consists of dividing the 58 sectors into two equally sized groups depending on the value of $\overline{\theta}_i$ and then estimating the regression

$$y_{f,i,t} = \alpha_f + \beta_t \times \mathbb{1}\left[\overline{\theta}_i < \text{median}\left(\overline{\theta}_i\right)\right] + \Gamma_t \times X_f + \epsilon_{f,i,t}, \tag{23}$$

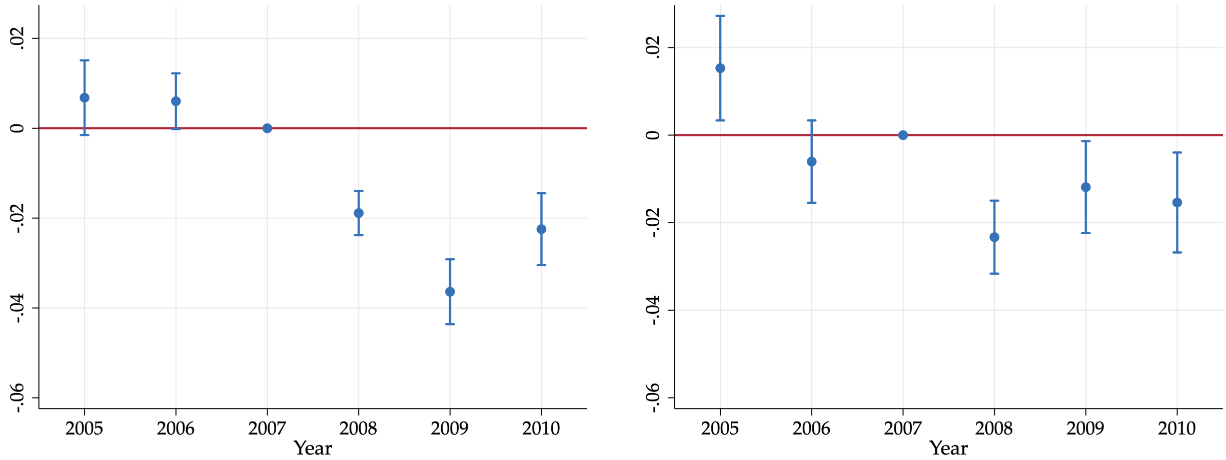
where $y_{f,i,t}$ is the dependent variable of interest (accounts payable or sales) of firm f operating in sector i at time t. The regression includes firm and time fixed effects, as well as firm and sectoral characteristics interacted with time fixed effects included to make sure that our results are not spurious.[33] The sample includes all firms between the years 2005–2015 which operate for at least 9 years, so that all firms in our balanced panel are included in the regression.

Figure 3 displays the estimates of β_t between 2005 and 2010.[34] Panel (a) presents the estimates for β_t when the dependent variable is the (log) of firms' sales. Consistent with the

[33]The controls include indicators for whether the average sales of the firm are larger than those of the median firm in the sample, whether the capital intensity proxied by the assets-to-sales ratio is higher than the median, whether the firm operates in manufacturing, and whether it operates in the service sector.

[34]We stop in 2010 because the Italian economy experienced a major debt crisis in 2011 which could confound our analysis, see Bocola (2016) and Arellano, Bai, and Bocola (2017).

theory, firms in sectors that were closer to the financially constrained region experienced a larger drop in sales after 2007. There is no indication of meaningful pre-trends as evidenced by the coefficients on years 2005–2006. Panel (b) of Figure 3 presents the same differential for firms' accounts payable. We can see that accounts payable for low-$\overline{\theta}_i$ sectors fell 2% relative to those of high-$\overline{\theta}_i$ sectors in the aftermath of the Great Recession. These results are consistent with Costello (2020) and other papers, who finds that suppliers suffering negative liquidity shocks reduce the amount of trade credit given to their customers and negatively impact their operations.



(a) Sales

(b) Accounts payable

Figure 3: Dynamic diff-in-diff results

Notes: These figures present the point estimates and 99% confidence intervals for β_t in (23). Panel (a) reports the results for log-sales as the dependent variable, while the dependent variable in panel (b) is the log of accounts payable. Standard errors are clustered at the firm level.

5.2 Calibration

We can classify the structural parameters into four groups: preference parameters $[\beta, \psi, \chi, \gamma]$, common financial and technology parameters, $[\delta, \sigma]$, sector-specific parameters, $\{\eta_i, \pi_i, N_i\}_i$, and parameters governing the stochastic process of the financial shock. For the quantitative analysis, we assume that θ_t follows a two-state Markov process $\theta_t \in \{\theta_L, \theta_H\}$. We interpret θ_L as the state of the financial sector in "normal times" and a switch to θ_H as a financial crisis. To be consistent with our data, a time period in the model corresponds to one year, and we consider 58 different sectors.

We set $\beta = 0.98$ and $\psi = 1.00$, standard values in the macroeconomic literature. In addition, we set $p(\theta_L|\theta_L) = 0.99$ to be consistent with the notion that financial crises are rare events in advanced economies. The remaining parameters are chosen simultaneously so that the model matches a set of sample moments computed using our dataset. Table 3 reports the calibrated parameters along with the empirical targets and their model counterpart in simulations. Below, we describe the sample moments and discuss heuristically which model parameters they help us discipline.

We target the ratio of accounts receivable over sales, expenditures on intermediate inputs/services over sales, and the $\text{HHI}_i^{\text{supplier}}$ for each of the sectors in 2007 and match it to the sample average in model simulations conditional on $\theta = \theta_L$. As we discussed previously, these variables identify the sector-specific parameters $\{\pi_i, \eta_i, N_i\}_i$. We also target the average ratio of accounts payable to sales in 2007. Given the other model parameters, this moment provides information about δ: a higher δ reduces the need for working capital needs by the final good firms in all sectors and, thus, trade credit. In our calibration, δ is equal to 0.54. A value of 3.5 for χ guarantees that the level of worked hours in our simulations equals one-third in normal times.

We use the behavior of the average ratio of firms' short-term bank loans over sales to discipline the stochastic process for θ_t. This ratio went from 0.16 in 2007 to 0.12 in 2008, and by 2011 it was back to 0.14. We choose the parameters of the stochastic process so that on average our model replicates this behavior when there is a switch from θ_L to θ_H. This yields $\theta_L = 0.78$, $\theta_H = 0.84$, and $p(\theta_H|\theta_H) = 0.86$.

The remaining parameters, γ and σ, have poorly measured empirical counterparts. To discipline the former, we target the peak-to-trough fall in average log sales during the Great Recession (0.12) and match it in the model to the average fall in firms' log sales conditional on a switch from θ_L to θ_H. As we explained in Section 4, γ partly hape the general equilibrium effects in the model, and it therefore control the sensitivity of output to the financial shock. To discipline σ, we include the diff-in-diff coefficient β_{2008} for log sales. Holding the other parameters fixed, changes in σ affect $\bar{\theta}_i$, and it thus affects the magnitude of this differential response. Our calibration yields $\gamma = 3$ and $\sigma = 5$.[35]

Besides having good in-sample fit, the model reproduces quite well the features of the distribution of accounts payable that are not targeted in the calibration. Table 4 reports the correlation across sectors between accounts payable and other key ratios in the data and in the model. The table shows that the quantitative model displays cross-sector correlations that are consistent with the data. The correlation between accounts payable and accounts receivable is 0.48 in the model, very close to its data counterpart of 0.59. The correlation between accounts payable and the share of intermediate inputs over sales is positive in the

[35] One may worry that our identification of σ is circular because the grouping of sectors underlying regression 23 is done using the calibrated value of σ. This classification, however, is not very sensitive to value of σ. Indeed, the estimates of β_{2008} remain equal to -0.02 for all $\sigma \in [4.5, 5.5]$.

Table 3: Model parameters and targeted moments

Parameter	Value	Moment	Data	Model
Mean(π_i)	0.52	Distribution of accounts receivable/sales		
Stdev(π_i)	0.14			
Mean(η_i)	0.81	Distribution of intermediate inputs/sales		
Stdev(η_i)	0.09			
Mean(N_i)	23.83	Distribution of HHI$^{\text{supplier}}$		
Stdev(N_i)	10.96			
δ	0.55	Mean(Accounts payable/sales)	0.23	0.23
χ	3.50	Mean(Worked hours/total hours)	0.33	0.33
θ_L	0.78	Mean(Bank loans/sales) in 2007	0.16	0.16
θ_H	0.84	Mean(Bank loans/sales) in 2008	0.12	0.12
$p(\theta_H \vert \theta_H)$	0.86	Mean(Bank loans/sales) in 2011	0.14	0.14
γ	3.00	% Fall in average firms' sales	0.12	0.12
σ	5.00	β_{2008} in eq. (23), log-sales as dep. var.	-0.02	-0.02

Notes: The table reports the numerical values of model parameters along with the empirical targets used in the calibration. The first six rows report statistics for the distribution of $\{\pi_i, \eta_i, N_i\}$ across the 58 sectors of our analysis. These parameters are chosen to replicate exactly the value of accounts receivable/sales, intermediate input costs/sales, and the HHI$^{\text{supplier}}$ for each of the 58 sectors. The other rows report the exact numerical values for the remaining parameters along with the moments used to discipline them. The sample moments in the model are computed using simulations.

data (0.21) and in the model (0.68), although it is substantially higher in the latter. The model implied correlation between accounts payable and the supplier HHI index is also close to the data (0.30 vs. 0.24). Finally, the model reproduces a positive correlation across sectors between accounts payable and bank loans.

5.3 The macroeconomic implications of trade credit

In Section 3, we focused on two main macroeconomic implications of trade credit. First, we showed that *on average*, trade credit relationships reduce the output costs of financial frictions via a credit multiplier effect. Second, we have seen that trade credit relationships can smooth or amplify the impact of financial shocks depending on suppliers' borrowing capacity. We now use the calibrated model to quantify these two aspects.

We start by measuring the size of the credit multiplier and quantifying its implications for output. For that purpose, Table 5 compares the average behavior of credit and output in the benchmark economy to those of two counterfactual economies. The first economy is identical in all respects to the benchmark, except that final good firms cannot issue trade

Table 4: Trade credit cross-sector correlation patterns

Moment	Data	Model
corr(payable/sales, receivable/sales)	0.59	0.48
corr(payable/sales, intermediate-inputs/sales)	0.21	0.68
corr(payable/sales, $\text{HHI}^{\text{supplier}}$)	0.24	0.30
corr(payable/sales, bank debt/sales)	0.35	0.59

Notes: The table reports cross-sector correlations of the accounts payable to sales ratio with different variables in the data and in the model. The data moments are computed for the year 2007. In the model, the statistics are averages from a long simulation conditioned on $\theta_t = \theta_L$.

credit to their suppliers, ($p_i^{\text{tc}} = 0$ for all i). In the second economy, final good firms cannot issue trade credit to suppliers *and* they are paid on the spot by their customers ($p_i^{\text{tc}} = 0$ and $\pi_i = 0$ for all i). We can think of the latter as the *spot economy* because none of the transactions between the different economic entities—intermediate good producers, final good producers, and households—involve the issuance of IOUs.

Let us start by comparing the benchmark with the $p_i^{\text{tc}} = 0$ economy. Total bank credit in the latter is on average 43% the size of bank credit in the benchmark. This is due to two effects. First, in the economy with trade credit, final good firms have higher revenues, so they can mechanically obtain more credit from banks.[36] Second, the suppliers discount their accounts receivable with banks in the economy with trade credit, something that doesn't happen in the counterfactual economy because suppliers do not borrow. This latter mechanism is quantitiatively sizable as credit backed by the accounts receivable of suppliers represents 27% of total bank credit to firms in our economy.[37]

Aside from increasing its quantity, the benchmark economy also has a better allocation of credit relative to the counterfactual. To see that, Table 5 also reports the fraction of bank credit that is directed toward the payments of productive inputs—the wages of workers. To that purpose, we compute the equilibrium wage payments by suppliers and subtract the cash that final good firms have available in the morning. We then scale this indicator by total bank credit to firms. A value of 1 means that all bank credit in the economy is allocated to the payment of productive inputs. We can see that in the benchmark economy, this ratio is indeed close to 1, while in the spot economy, most of the bank credit is used to pay for suppliers' rents. The combination of these two factors—less credit and a worse allocation—implies that output in the $p_i^{\text{tc}} = 0$ economy is 60% that of the benchmark, as

[36] In both economies, firms' borrowing limit is a fraction $(1 - \theta_t)$ of their afternoon revenues.

[37] This figure compares quite well with the untargeted data counterpart of 40% computed using the Italian credit registry and the 30% reported by Caglio, Darst, and Kalemli-Ozcan (2021) for the US economy, see the discussion of Section 3.1.

Table 5: Quantifying the credit multiplier

		Benchmark	$p_i^{\text{tc}} = 0$	$p_i^{\text{tc}} = 0$ and $\pi_i = 0$
Bank credit		1.00	0.43	0.33
	To final good firms	0.73	0.43	0.33
	To suppliers	0.27	0.00	
	% allocated to wages	0.96	0.02	0.03
Output		1.00	0.60	0.86

Notes: This table reports bank credit and output in the benchmark economy and in the two counterfactual economies. The figure reports time averages conditional on the economy being in good times ($\theta_t = \theta_L$). We normalize the bank credit and output by their value in the benchmark economy.

the last row of Table 5 shows.

In the counterfactual we just discussed, final good firms cannot issue IOUs to their suppliers but they do accept IOUs from their customers, the households. In the last column of Table 5, we report what happens to credit and output in an economy in which households also pay on the spot, $\pi_i = 0$ for all i. We can see that output is higher while credit is lower relative to the previous counterfactual. This happens because final good firms receive more cash in the morning when $\pi_i = 0$, so their need for working capital is smaller—something that mitigates the economic costs of financial shocks. Qualitatively, however, the comparison with the benchmark is similar to that of the previous counterfactual: the spot economy features substantially less bank credit relative to the benchmark and 14% lower output.

The second quantitative exercise consists of assessing how trade credit shaped the response to a financial shocks similar in magnitude to that of the Great Recession. To do so, we study the response of the benchmark economy to a tightening of aggregate financial conditions—a switch from θ_L to θ_H—and compare it to what happens in the two counterfactual economies.

Figure 4 reports the response of output to the financial shock in the benchmark economy and in the two counterfactuals. In the benchmark economy (solid line), output falls 11% on impact, as in the Italian data. This is not surprising, since the fall in output conditional on a financial shock was part of the calibration targets. In the two counterfactual economies, the output effects of the same shock are much smaller: in the $p_i^{\text{tc}} = 0$ economy, output falls by 7.7% on impact, while in the spot economy, it falls by only 5.9%. This means that the presence of trade credit substantially *amplified* the macroeconomic implications of financial shocks during the Great Recession—accounting for between 30% to 45% of the total response, depending on which counterfactual we consider in the comparison.

This large amplification is due to the fact that the supply chains in the economy with trade credit are substantially more levered relative to what happens in the counterfactual spot economy, as exemplified by the fact that roughly one-third of total bank credit is backed by accounts receivable. This makes the economy way more sensitive to financial shocks, as we explained in Section 3.2.

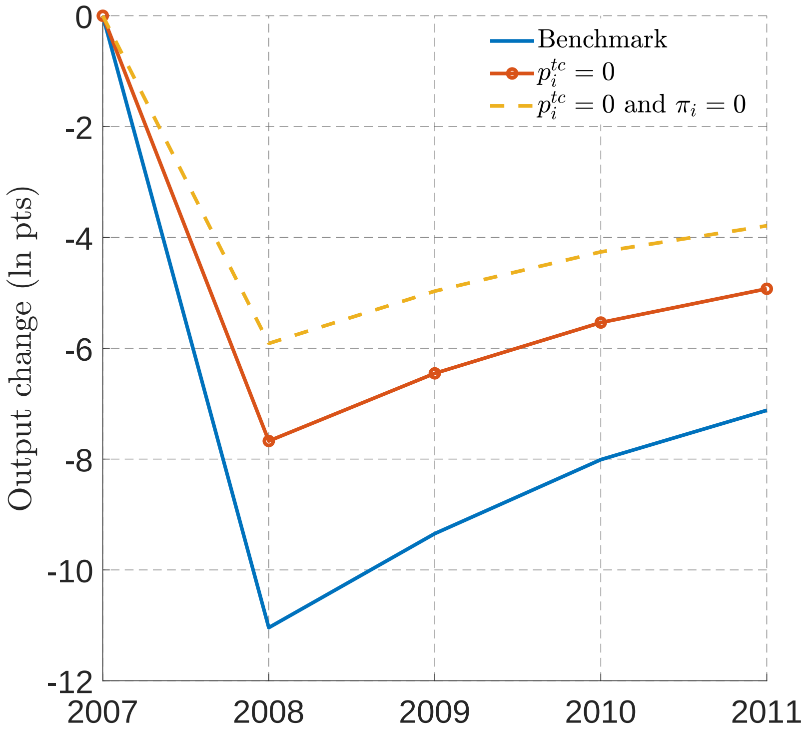


Figure 4: The implications of trade credit for the Great Recession

Note: This figure displays the response of output to a financial shock in the benchmark economy (solid line), the counterfactual economy with $p_i^{\text{tc}} = 0$ (line with circles) and the counterfactual economy with $p_i^{\text{tc}} = 0$ and $\pi_i = 0$ (dashed line).

5.4 The effects of corporate subsidies

In this section, we study the effectiveness of corporate subsidies in alleviating financial constraints and stimulating output during financial crises. It is immediate to see that such subsidies raise output in our environment. Our focus, however, is on studying which firms would benefit the most: would subsidies be more effective in stimulating output if they were directed toward downstream or upstream firms? We start by analyzing this question in the special case of Section 3 and show that the subsidy is more effective when targeted toward the financially constrained supplier. We then move on to the quantitative model and compare alternative types of interventions.

To illustrate the difference between a lump-sum subsidy to final good producers and one to suppliers, we first consider the special case economy studied in Section 3, which features a single production line with a single supplier. We further assume that there are no aggregate shocks, that all final goods are received at the end of the period ($\delta = 0$), and that the degree of financial frictions is such that the borrowing constraint of the monopolist binds in the steady state absent corporate subsidies ($\theta > \bar{\theta}$). The problem of the supplier with lump-sum subsidies is as follows:

$$
\begin{aligned}
V(J) = \max_{\{x, p^{\mathrm{s}}, p^{\mathrm{tc}}, J'\}} \quad & \left(p^{\mathrm{s}} + p^{\mathrm{tc}} - Wx\right) + T_s + \beta V(J'), \\
\text{s.t.} \qquad & p^{\mathrm{s}} \le (1-\theta)x^{\eta} + T_f, \\
& Wx - p^{\mathrm{s}} \le (1-\theta)p^{\mathrm{tc}} + T_s, \\
& p^{\mathrm{tc}} \le \beta J', \\
& p^{\mathrm{s}} + p^{\mathrm{tc}} \le x^{\eta} + T_f, \\
& J = x^{\eta} - (p^{\mathrm{s}} + p^{\mathrm{tc}}) + \beta J',
\end{aligned}
$$

where T_f is the periodic lump-sum subsidy to final good firms, and T_s is the one to the monopolist. Notice that J, the customer value of its relationship with the supplier, does not include T_f because the subsidy is paid to the customer independently of its relationship with the supplier.

The following proposition characterizes the effects of the two subsidies in steady state when the supplier is financially constrained.

Proposition 4. *Suppose that $\theta > \bar{\theta}$ where $\bar{\theta}$ is defined implicitly by $(1-\bar{\theta})(1+\beta\bar{\theta}) = \eta$. Then, the effects of corporate subsidies on employment in the steady state around the point without corporate subsidies are given by*

$$
\left.\frac{\partial x}{\partial T_s}\right|_{T_s = T_f = 0} = \frac{1}{(1-\eta)W}, \tag{24}
$$

and

$$
\left.\frac{\partial x}{\partial T_f}\right|_{T_s = T_f = 0} = \frac{1-\beta(1-\theta)}{(1-\eta)W}. \tag{25}
$$

Proposition 4 states that when the monopolist's borrowing constraint binds, a subsidy to the monopolist is more effective in stimulating output relative to a subsidy to the final good producer. While both types of subsidies stimulate employment, a corporate subsidy to final good firms is only a fraction $[1-\beta(1-\theta)]$ as effective.

To understand this result, let's substitute the borrowing constraint of the final good producers with that of the supplier, both binding around the *laissez-faire* steady state, to obtain

$$Wx = (1-\theta)x^{\eta} + (1-\theta)p^{\text{tc}} + T_s + T_f.$$

We can then use this expression to study the effects of corporate subsidies on output. The two subsidies enter this expression in a similar fashion, so they will end up having different output effects only if they have differential effects on p^{tc}. This is precisely what happens, with T_f crowding out trade credit more than T_s. Indeed, substituting the borrowing constraint of the final good producer into the expression for J, we obtain that in steady state

$$p^{\text{tc}} = \beta J = \beta[\theta x^{\eta} - T_f].$$

Holding x constant, a one dollar subsidy to final good firms reduces one-for-one the value of their relationship with the supplier, and it therefore depresses the steady state level of trade credit by β dollars. This effect doesn't happen when the subsidy is given to the supplier. So, T_f ends up crowding out trade credit more relative to T_s, explaining why the latter is a better tool for stimulating output.

We now return to the quantitative model. We assume that during a financial crisis, the government has limited resources to support financially constrained firms.[38] We denote by T_i^{supplier} and T_i^{final} the lump-sum corporate subsidies to suppliers and final good producers in production line i, respectively. The total sum of these subsidies is levied as a lump-sum tax on households in the economy. We assume that these subsidies are given at the start of the period as long as $\theta = \theta_H$—that is, as long as the economy remains in a financial crisis.

In addition to directly changing the profits of final good producers and suppliers, corporate subsidies affect the borrowing constraints of both types of firms. In Appendix A.1.1, we lay out the supplier's problem with corporate subsidies. We assume that the size of government subsidies is equal to 1% of GDP in normal times. We consider five configurations of corporate subsidies: (i) a uniform subsidy to all firms, (ii) a uniform subsidy to all producers, (iii) a uniform subsidy to all suppliers, (iv) a uniform subsidy to all sectors with $\theta > \bar{\theta}_i$, and (v) a uniform subsidy to all suppliers in sectors with $\theta > \bar{\theta}_i$. Table 6 presents the results.

The first row of Table 6 shows that a uniform subsidy to all firms during the Great Recession raises output by about 0.9%. That is, instead of output declining by 11% in 2008, a uniform subsidy to all firms would reduce the decline to 10.1%. The second and third

[38]Note that we assume that there is perfect information so that the government can perfectly observe which sectors are financially constrained. Dávila and Hébert (2023) study the optimal design of corporate taxation with private information so that the government cannot directly observe which firms are financially constrained. Unlike their paper, our focus is on where on the supply chain a corporate subsidy is more effective when trade credit is available.

Table 6: The effects of corporate subsidies during the Great Recession

Subsidies	Output gain	Effectiveness
Uniform across firms	0.91	1
Uniform across producers	0.85	0.93
Uniform across suppliers	0.98	1.07
Uniform across firms with $\bar{\theta}_i < \theta_H$	0.98	1.08
Uniform across suppliers with $\bar{\theta}_i < \theta_H$	1.05	1.15

Notes: This table reports the counterfactual increase in output (in log points) during 2008 relative to our benchmark specification for different allocations of subsidies across firms. The total amount of subsidies is set to be 1% of output in normal times. Effectiveness represents the relative contribution to output of the subsidies scheme with respect to a uniform subsidy to all firms. The larger the effectiveness, the larger is the contribution to output.

rows display the effects of corporate subsidies when given uniformly to only producers and only suppliers, respectively. As Proposition 4 suggests, a uniform subsidy to suppliers is more effective in stimulating output during a financial crisis. While a uniform subsidy to producers raises output by 0.85 log points, a uniform subsidy to suppliers raises output by 0.98 log points. That is, a uniform subsidy to suppliers in the economy is about 15% more effective than a uniform subsidy to all producers and about 7% more effective than a uniform subsidies to all firms in the economy.

The final two rows of Table 6 display the effects of corporate subsidies when targeted toward sectors which are more likely to be financially constrained. Overall, there are 53 sectors in the economy with $\bar{\theta}_i < \theta_H$. Corporate subsidies are more effective at stimulating output when targeted toward these sectors. A uniform subsidy to all firms in these financially constrained sectors raises output by 0.98 log points—an increase in 7% relative to an untargeted subsidy configuration toward all sectors. A uniform subsidy toward only suppliers in financially constrained sectors has the largest effect on output from the configurations we've analyzed. Under such subsidy configuration, output goes up by 1.05 log points—15% more effective than a uniform subsidy to all firms in all sectors in the economy.

Our paper is not the first to study the effectiveness of corporate subsidies in stimulating output in the context of production networks. Liu (2019) shows that optimal corporate subsidies in a production network should be targeted toward sectors that are more *central* to market imperfections—sectors that supply a disproportionate fraction of output to other sectors with severe market imperfections. He finds that these sectors are typically upstream sectors. Glode and Opp (2021) studies the effectiveness of corporate subsidies in a production chain when firms can default on their trade credit in equilibrium. They find that corporate subsidies can be more effective when targeted toward downstream producers, as such subsidies can help prevent *default waves*. Relative to these papers, our analysis sheds light on another motive that shapes the design of optimal corporate subsidies—their effects on trade-credit linkages among firms.

6 Conclusion

We have proposed an equilibrium model to explain the prevalence of trade credit as a form of short-term financing for companies around the world and used it to understand the macroeconomic implications of this phenomenon. In our theory, trade credit is enforced in equilibrium by reputational forces, as customers have an incentive to repay their suppliers out of fear of losing that relationship. This mechanism allows the economy to increase credit provided by the financial system because suppliers can enforce these IOUs and at the same time discount these claims with financial institutions to obtain liquidity. We provide evidence consistent with this theory and fit the model to Italian data in order to quantify the macroeconomic implications of the credit multiplier. We show that this process allows the economy to support 16% more output on average, but it also makes the economy more vulnerable to financial shocks, as the presence of trade credit substantially amplified the output costs of the Great Recession.

We believe that this framework could be used to address a number of important questions. For example, we could apply it to study the role of trade credit relationships in the propagation of firm-specific shocks throughout the production network. In addition, the forward-looking aspect of these relationships makes them particularly vulnerable to self-fulfilling confidence crises, something that could rationalize the sudden disruptions in the payments' chain observed for certain countries during the Great Recession. We plan to address these and other exciting questions in future research.

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Appendix

A.1 Full model - recursive formulation

In Section 2, we've presented the problem of the intermediate-good producer in sequence form. In this section, we lay out the recursive formulation of the problem. We will use the recursive formulation to derive optimality conditions and to prove the different propositions in the paper.

Before writing down the recursive formulation, it is useful to briefly discuss the state variables in the firm's problem. First, the exogenous state variable is θ, the degree of financial frictions in the economy. In addition, each intermediate-good producer carries over a promise made in the previous period to final good firms. This promise guarantees the final good producer a discounted surplus of the match, $\tilde{J}_i^j$. We study a symmetric equilibrium, under which the promise made by suppliers in production lines in sector i are all identical and we can denote them by $\tilde{J}_i$. The aggregate state of the economy therefore consists of θ as well as the distribution of $\tilde{J}_i$ across all sectors, which we denote by Ω. We will denote the wage that arises in general equilibrium by $w(\theta, \Omega)$ and the revenue of final good firm i by $rev_i(\{x_n\}_{n=1}^{N_i}, \theta, \Omega)$. The latter is given by

$$rev_i(\{x_n\}_{n=1}^{N_i}, \theta, \Omega) = P(\theta, \Omega)C(\theta, \Omega)^{\frac{1}{\gamma}} \left(k^{1-\eta_i} \left\{ \left[\sum_{n=1}^{N_i} x_n^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\}^{\eta_i} \right)^{\frac{\gamma-1}{\gamma}} \tag{A.1}$$

with $P(\theta, \Omega)$ and $C(\theta, \Omega)$ denoting the aggregate price index and consumption in the economy.

The recursive problem of the intermediate-good producer supplying goods to final good firms in sector i is given by

$$\begin{aligned}
V\left(\tilde{J}(\theta), \theta, \Omega\right) = \max_{p_j^s, p_j^{tc}, x_j, \tilde{J}'(\theta')} \; & p_j^s + p_j^{tc} - w(\theta, \Omega)x_j + \beta \mathbb{E}\left[V\left(\tilde{J}'(\theta'), \theta', \Omega'(\theta')\right)\right] \\
s.t. \quad & p_j^s + \sum_{n \neq j} p_n^s \leq [1 - \theta(1 - \delta(1 - \pi_i))] \, rev_i(\{x_j, \vec{x}_{-j}\}, \theta, \Omega), && [\mu] \\
& p_j^{tc} \leq \beta \mathbb{E}\left[\tilde{J}'(\theta')\right], && [\gamma] \\
& w(\theta, \Omega)x_j - p_j^s \leq (1 - \theta)p_j^{tc}, && [\iota] \\
& \tilde{J}(\theta) = rev_i(\{x_j, \vec{x}_{-j}\}, \theta, \Omega) - rev_i(\{0, \vec{x}_{-j}\}, \theta, \Omega) - (p_j^s + p_j^{tc}) + \beta \mathbb{E}_\theta\left[\tilde{J}'(\theta')\right], && [\lambda] \\
& p_j^s + p_j^{tc} + \sum_{n \neq j} (p_n^s + p_n^{tc}) \leq rev_i(\{x_j, \vec{x}_{-j}\}, \theta, \Omega), && [\rho]
\end{aligned}$$

where the red letters correspond to the Lagrange multipliers associated with each equation. The first-order conditions are given by

$$[p^s] \qquad 1+\iota = \mu+\lambda+\rho, \tag{A.2}$$

$$[p^{tc}] \qquad 1+(1-\theta)\iota = \gamma+\lambda+\rho, \tag{A.3}$$

$$[x] \qquad [(1-\theta\left(1-\delta(1-\pi_i)\right))\mu+\rho+\lambda]\,\frac{\partial rev_i(\{x_j,\vec{x}_{-j}\},\theta,\Omega)}{x_j} = w(\theta,\Omega)(1+\iota), \tag{A.4}$$

$$[J'(\theta')] \qquad \beta P(\theta'|\theta)V_J\left(J'\left(\theta'\right),\theta'\right) = -\beta P(\theta'|\theta)\left(\gamma+\lambda\right), \tag{A.5}$$

The envelope condition is given by

$$V_{\tilde{J}(\theta)}\left(\tilde{J}\left(\theta\right),\theta\right) = -\lambda(\theta), \tag{A.6}$$

Combining the two sets of conditions above we obtain the following optimality condition

$$\lambda'(\theta') = \lambda(\theta)+\gamma(\theta). \tag{A.7}$$

In the symmetric equilibrium, we have that

$$rev_i(x,\theta,\Omega) = P(\theta,\Omega)C(\theta,\Omega)^{\frac{1}{\gamma}}k_i^{(1-\eta)\frac{\gamma-1}{\gamma}}N_i^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}x^{\eta\frac{\gamma-1}{\gamma}},$$

and[39]

$$\frac{\partial rev_i(x,\theta,\Omega)}{\partial x_j} = \frac{\gamma-1}{\gamma}\eta P(\theta,\Omega)C(\theta,\Omega)^{\frac{1}{\gamma}}k_i^{(1-\eta)\frac{\gamma-1}{\gamma}}N_i^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}-1}x^{\eta\frac{\gamma-1}{\gamma}-1}. \tag{A.8}$$

A.1.1 The recursive problem with corporate subsidies

We assume that the government uses lump-sum subsidies to support financially constrained firms during financial crises. In particular, $N_i T_i^{\text{final}}$ denotes the total lump-sum subsidy to final-good producers and T_i^{supplier} the lump-sum subsidies to each supplier in production line i. The recursive problem of the intermediate-good producer supplying goods to final good firms is then given by

[39] Note that the second expression is not obtained by differentiating the first with respect to x, but rather by imposing symmetry in the expression of $\frac{\partial rev_i(\{x_j,\vec{x}_{-j}\})}{\partial x_j}$.

$$
\begin{aligned}
V\left(\tilde{J}(\theta),\theta,\Omega\right) = & \max_{p_j^s,p_j^{tc},x_j,\tilde{J}'(\theta')} p_j^s + p_j^{tc} - w(\theta,\Omega)x_j + T_i^{\text{supplier}}\mathbb{1}\left(\theta=\theta^H\right) + \beta\mathbb{E}\left[V\left(\tilde{J}'(\theta'),\theta',\Omega'(\theta')\right)\right] \\
s.t. \quad & p_j^s + \sum_{n\neq j} p_n^s \leq \left[1-\theta\left(1-\delta(1-\pi_i)\right)\right] rev_i(\{x_j,\vec{x}_{-j}\},\theta,\Omega) + N_i T_i^{\text{final}}\mathbb{1}\left(\theta=\theta^H\right), && [\mu] \\
& p_j^{tc} \leq \beta\mathbb{E}\left[\tilde{J}'(\theta')\right], && [\gamma] \\
& w(\theta,\Omega)x_j - p_j^s \leq (1-\theta)p_j^{tc} + T_i^{\text{supplier}}\mathbb{1}\left(\theta=\theta^H\right), && [\iota] \\
& \tilde{J}(\theta) = rev_i(\{x_j,\vec{x}_{-j}\},\theta,\Omega) - rev_i(\{0,\vec{x}_{-j}\},\theta,\Omega) - (p_j^s+p_j^{tc}) + \beta\mathbb{E}_\theta\left[\tilde{J}'(\theta')\right], && [\lambda] \\
& p_j^s + p_j^{tc} + \sum_{n\neq j}(p_n^s+p_n^{tc}) \leq rev_i(\{x_j,\vec{x}_{-j}\},\theta,\Omega) + N_i T_i^{\text{final}}\mathbb{1}\left(\theta=\theta^H\right), && [\rho] \\
& \tilde{J}'(\theta') \geq 0. && [\zeta(\theta')]
\end{aligned}
$$

A.2 Proofs

Proof of Proposition 1

Proof. In the steady state, $\lambda' = \lambda$ so that equation (16) implies $\kappa = 0$. Combining equations (14) and (15), we obtain $\mu = \theta\iota$. In the steady state, the promise keeping constraint boils down to

$$
J = \frac{1}{1-\beta}\left(x^\eta - p^s - p^{tc}\right).
$$

Consider first the case where $\mu = 0$. In that case $\iota = 0$ and the first order condition with respect to x implies

$$
x = \underbrace{\left(\frac{\eta}{W}\right)^{\frac{1}{1-\eta}}}_{x_{fb}}.
$$

From the promise keeping constraint, we obtain that $p^s + p^{tc} = x_{fb}^\eta - (1-\beta)J$. For a given level of J, the borrowing constraints do not bind if

$$
\begin{aligned}
& J \geq \tilde{\theta}(1-\delta)x_{fb}^\eta, \\
& J \leq \frac{1}{(1-\tilde{\theta})(1-\beta)}\left[\left((1-\tilde{\theta})(1+\tilde{\theta}(1-\delta)) + \delta\tilde{\theta}\right)x_{fb}^\eta - Wx_{fb}\right],
\end{aligned}
$$

where the first constraint is the trade credit constraint after using the promise keeping constraint together with the borrowing constraint of the final-good firm. The second constraint is the borrowing constraint of the supplier. Therefore, any level of J that satisfies both conditions, if such exists, can be sustained as a steady state. We will restrict attention to the lower bound of such support ($J = \tilde{\theta}(1-\delta)x_{fb}^\eta$), which guarantees the supplier the most surplus in the steady state.[40] This level of J is supported by $p^{tc} = \beta\tilde{\theta}(1-\delta)x^\eta$ and

[40]Note that such equilibrium selection does not affect output in the economy as $x = x^{fb}$ when $\iota = \mu = 0$

$p^s = (\delta + (1-\delta)(1-\tilde{\theta}))x^\eta$.

Depending on parameter values, there could potentially be no level of J such that both borrowing constraints do not bind in the steady state. That is, for some parameters, the borrowing constraints bind in the steady state. Such case occurs when

$$(1-\tilde{\theta})(1-\beta)\tilde{\theta}(1-\delta)x_{fb}^\eta > \left[\left((1-\tilde{\theta})(1+\tilde{\theta}(1-\delta)) + \delta\tilde{\theta}\right) - \eta\right] x_{fb}^\eta,$$

where we've used the definition of x_{fb} to replace the wage bill. Rearranging, we obtain that the borrowing constraints bind in the steady state if and only if

$$\eta > \delta + (1-\tilde{\theta})(1-\delta)(1+\beta\tilde{\theta}).$$

Note that the RHS is decreasing in $\tilde{\theta}$. Let $\bar{\theta}$ be the level of $\tilde{\theta}$ which makes the equation above hold with equality. For any $\tilde{\theta} \geq \bar{\theta}$, the condition above is satisfied and both borrowing constraints bind in the steady state. In this case, we have that $p^s = (\delta + (1-\delta)(1-\tilde{\theta}))x^\eta$ and the promise keeping constraint implies that $(1-\beta)J = \tilde{\theta}(1-\delta)x^\eta - p^{tc}$. The supplier's borrowing constraint is

$$(1-\tilde{\theta})(1-\beta)J = (\delta + (1-\delta)(1-\tilde{\theta})(1+\tilde{\theta}))x^\eta - Wx.$$

For the trade credit constraint to be satisfied, we must have $J \geq \tilde{\theta}(1-\delta)x^\eta$. As in the unconstrained case, we restrict attention to the lowest level of J which can be supported in equilibrium. This level of J is supported by the upper limit of trade credit, $p^{tc} = \beta J = \beta\tilde{\theta}(1-\delta)x^\eta$. Plugging into the supplier's borrowing constraint we obtain:

$$Wx = (\delta + (1-\delta)(1-\tilde{\theta}))x^\eta + (1-\tilde{\theta})\beta\tilde{\theta}(1-\delta)x^\eta,$$

which implies that when the $\tilde{\theta} > \bar{\theta}$,

$$x = \left[\frac{\delta + (1-\delta)(1-\tilde{\theta})(1+\beta\tilde{\theta})}{W}\right]^{\frac{1}{1-\eta}}$$

□

Proof of Proposition 2

Proof. We will show that when $\theta < \bar{\theta}$, the equilibrium level of output does not change with θ. We conjecture that

regardless of the level of J.

$$J(\theta) = \theta(1-\delta)\left(\frac{\eta}{W}\right)^{\frac{\eta}{1-\eta}},$$

and that the borrowing constraints as well as the trade credit constraint are not binding so that $\mu(\theta) = \omega(\theta) = \kappa(\theta) = \iota(\theta) = 0$. From equation (13), this implies

$$x = \left(\frac{\eta}{W}\right)^{\frac{1}{1-\eta}}.$$

We want to verify that our conjecture for J_θ constitutes an equilibrium, in which the borrowing constraints as well as the trade credit constraint are not binding. We set

$$p^s = \delta + (1-\delta)(1-\theta)\left(\frac{\eta}{W}\right)^{\frac{\eta}{1-\eta}}, \tag{A.9}$$

$$p^{tc} = \beta J = \beta\theta(1-\delta)\left(\frac{\eta}{W}\right)^{\frac{\eta}{1-\eta}}. \tag{A.10}$$

When the spot price and trade credit price are equal to these values, we confirm that the promise keeping equation holds with equality for the value of $J(\theta)$ we conjectured. We then check whether the supplier borrowing constraint (10) is satisfied:

$$W\left(\frac{\eta}{W}\right)^{\frac{1}{1-\eta}} - (\delta + (1-\delta)(1-\theta))\left(\frac{\eta}{W}\right)^{\frac{\eta}{1-\eta}} \leq \beta\theta(1-\theta)(1-\delta)\left(\frac{\eta}{W}\right)^{\frac{\eta}{1-\eta}},$$

Rearranging we obtain

$$\eta \leq \delta + (1-\delta)(1-\theta)(1+\beta\theta). \tag{A.11}$$

Note that the RHS is decreasing in θ, and that for $\bar{\theta}$ the equation above holds with equality. So for all $\theta \leq \bar{\theta}$, this inequality holds, and we have that the supplier's borrowing constraint is not binding. Thus, we confirm that our conjecture for $J(\theta)$ consists of an equilibrium in which the borrowing constraints as well as the trade credit constraint are not binding.

Since the value of x is independent of θ, any change to θ which doesn't move it above $\bar{\theta}$, does not lead to a change in x. That is, output does not change in response to a small shock that raises θ as long as $\theta < \bar{\theta}$.

□

Proof of Proposition 3

Proof. We shall study the deterministic steady state when the wage level is W and aggregate consumption is C. In the steady state, the promise-keeping values as well as the Lagrange multipliers do not change over time. Equation (A.7) then implies $\gamma = \zeta = 0$, and combining equations (A.2)–(A.3), we get $\mu = \theta\iota$ and $\rho + \lambda = 1$. In a symmetric equilibrium, all suppliers in a production line will supply the same quantity of intermediate inputs. To ease notation, we will use $rev_i(x)$ to denote $rev_i(\{x, \dots, x\})$. In equilibrium we obtain

$$rev_i(x) = C^{\frac{1}{\gamma}} k_i^{(1-\eta)\frac{\gamma-1}{\gamma}} N_i^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}} x^{\eta\frac{\gamma-1}{\gamma}},$$

and[41]

$$\frac{\partial rev_i(x)}{\partial x_j} = \frac{\gamma-1}{\gamma}\eta P C^{\frac{1}{\gamma}} k_i^{(1-\eta)\frac{\gamma-1}{\gamma}} N_i^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}-1} x^{\eta\frac{\gamma-1}{\gamma}-1}. \tag{A.12}$$

Finally, in the steady state of a symmetric equilibrium, the promise keeping constraint is:

$$\tilde{J}(\theta) = A_i rev_i - \frac{1}{N_i} p^s - \frac{1}{N_i} p^{tc} + \beta \mathbb{E}_\theta[\tilde{J}'(\theta')], \tag{A.13}$$

where

$$A_i \equiv 1 - \left(\frac{N_i - 1}{N_i}\right)^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}.$$

Let's first analyze an equilibrium in which $\iota = 0$ and study the conditions under which such equilibrium exists. Using Equation (A.4) and Equation (A.12), we can solve for x_i and obtain:

$$x_i = \underbrace{\left[\frac{k^{(1-\eta_i)\frac{\gamma-1}{\gamma}} \eta_i \frac{\gamma-1}{\gamma} C^{\frac{1}{\gamma}}}{W N_i^{1-\eta_i\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}}\right]^{\frac{\gamma}{\eta_i + (1-\eta_i)\gamma}}}_{\equiv x_i^{unc}}. \tag{A.14}$$

Let's further conjecture that $p^{tc} = 0$ and $\tilde{J} = 0$. From the promise keeping constraint it then follows that

$$p^s = N_i A_i rev_i(x_i^{\text{unc}}).$$

The final producer's borrowing constraint must hold for this allocation to be sustained in

[41]Note that the second expression is not obtained by differentiating the first with respect to x, but rather by imposing symmetry in the expression of $\frac{\partial rev_i(\{x_j, \vec{x}_{-j}\})}{\partial x_j}$.

equilibrium, so the above characterizes the solution as long as

$$N_i A_i \leq 1 - \theta\left(1 - \delta(1-\pi_i)\right) \iff \theta \leq \underbrace{\frac{1 - N_i A_i}{1 - \delta(1-\pi_i)}}_{\underline{\theta}_i}. \tag{A.15}$$

That is, if $\theta \leq \underline{\theta}_i$ then the borrowing constraints for both the final-good producer as well as the supplier do not bind in the steady state allocations and $x_i = x_i^{unc}$. For $\theta > \underline{\theta}_i$, it is still possible that x_i^{unc} can be supported in equilibrium using trade credit. The borrowing limit of the final-good producer yields

$$p_i^s = [1 - \theta(1 - \delta(1-\pi_i))] rev_i(x_i^{\text{unc}}). \tag{A.16}$$

To determine p_i^{tc}, note that it should satisfy

$$p_i^{tc} \leq N_i \beta \tilde{J} = N_i \beta \left[A_i - \frac{1 - \theta(1 - \delta(1-\pi_i))}{N_i} \right] rev_i(x_i^{unc}).$$

To support x_i^{unc} in equilibrium, the supplier's borrowing constraint must hold so that

$$W x_i^{unc} \leq \frac{p_i^s}{N_i} + (1-\theta)\frac{p_i^{tc}}{N_i}.$$

Plugging in the upper limit for p_i^{tc} and the expression for p_i^s, we have the following condition

$$W N_i x_i^{unc} \leq [1 - \theta(1 - \delta(1-\pi_i))] rev_i(x_i^{\text{unc}}) + (1-\theta) N_i \beta \left[A_i - \frac{1 - \theta(1 - \delta(1-\pi_i))}{N_i} \right] rev_i(x_i^{unc}). \tag{A.17}$$

Note that the RHS is decreasing in θ. Let $\overline{\theta}_i$ be the level of θ such that the expression above holds with equality. If $\theta \in (\underline{\theta}_i, \overline{\theta}_i]$ then the supplier's borrowing constraint doesn't bind and $x = x^{unc}$ is the equilibrium allocation with a positive level of trade credit in the economy.

Finally, consider the case in which $\theta > \overline{\theta}_i$. In such case, x_i^{unc} cannot be supported in equilibrium and μ and ι are both strictly positive. In this case, the supplier's borrowing constraint binds, so that $x_i = x_i^{con}$ defined implicitly as

$$W N_i x_i^{\text{con}} = [1 - \theta(1 - \delta(1-\pi_i))] rev_i(x_i^{\text{con}}) + (1-\theta) N_i \beta \left[A_i - \frac{1 - \theta(1 - \delta(1-\pi_i))}{N_i} \right] rev_i(x_i^{\text{con}}),$$

with

$$p_i^s = [1 - \theta(1 - \delta(1-\pi_i))] rev_i(x_i^{\text{con}}),$$

and

$$p_i^{tc} = N_i\beta\left[A_i - \frac{1-\theta(1-\delta(1-\pi_i))}{N_i}\right] rev_i(x_i^{con}).$$

□

Proof of Lemma 1

Proof. From proposition 4, we know that if $\theta > \underline{\theta}_i$

$$\frac{p_i^{tc}}{rev_i} = N_i\beta\left[A_i - \frac{1-\theta(1-\delta(1-\pi_i))}{N_i}\right]$$

where

$$A_i \equiv 1 - \left(\frac{N_i-1}{N_i}\right)^{\eta_i\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}$$

Then

$$\frac{\partial\frac{p_i^{tc}}{rev_i}}{\partial\eta_i} = -\beta N_i\left(\frac{N_i-1}{N_i}\right)^{\eta_i\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}\log\left(\frac{N_i-1}{N_i}\right) > 0 \text{ since } \frac{N_i-1}{N_i} < 1 \text{ and } \sigma > \gamma > 1$$

$$\frac{\partial\frac{p_i^{tc}}{rev_i}}{\partial\pi_i} = \beta\delta > 0$$

Finally, provided that $\eta_i\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma} < 1$

$$\frac{\partial\frac{p_i^{tc}}{rev_i}}{\partial N_i} = \beta\left\{1 - \left(\frac{N_i-1}{N_i}\right)^{\eta_i\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}} - \frac{1}{N_i}\eta_i\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}\left(\frac{N_i-1}{N_i}\right)^{\eta_i\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}\right\} < 0 \text{ for all } N_i > 1$$

□

Proof of Proposition 4

Proof. This proof is very similar to the proof to Proposition 1. Note that the corporate subsidies do not alter the optimality conditions but only the constraints. In the steady state, $\lambda' = \lambda$ so that equation (16) implies $\kappa = 0$. Combining equations (14) and (15), we obtain $\mu = \theta\iota$. In the steady state, the promise keeping constraint boils down to

$$J = \frac{1}{1-\beta}\left(x^{\eta} - \tilde{p}^{s} - p^{tc} - T_f\right).$$

Since $\theta > \bar{\theta}$, we know that the monopolist borrowing constraints binds in the steady state when $T_f = T_s = 0$ (see proof to Proposition 1). We have that $p^s = (1-\tilde{\theta})x^\eta$ and the promise keeping constraint implies $(1-\beta)J = \theta x^\eta - p^{tc} - T_f$. The supplier's borrowing constraint is

$$(1-\theta)(1-\beta)J = \left[(1+\theta)(1-\theta)x^\eta - Wx + \theta T_f + T_s\right].$$

For the trade credit constraint to be satisfied, we must have $J \geq \theta x^\eta$. We restrict attention to the lowest level of J which can be supported in equilibrium. This level of J is supported by the upper limit of trade credit, $p^{tc} = \beta J = \beta\theta x^\eta - \beta T_f$. Plugging $J = \theta x^\eta - T_f$ into the supplier borrowing constraint we obtain:

$$(1-\theta)(1-\beta)\left(\theta x^\eta - T_f\right) = \left[(1+\theta)(1-\theta)x^\eta - Wx + \theta T_f + T_s\right].$$

Rearranging we obtain that

$$(1+\beta\theta)(1-\theta)x^\eta + (1-\beta(1-\theta))\,T_f + T_s = Wx$$

Totally differentiating with respect to x, T_f, and T_s, we have

$$\frac{1}{x}\eta(1+\beta\theta)(1-\theta)x^\eta dx + (1-\beta(1-\theta))\,dT_f + dT_s = Wdx,$$

rearranging we have

$$(1-\beta(1-\theta))\,dT_f + dT_s = \frac{1}{x}\left(Wx - \eta(1+\beta\theta)(1-\theta)x^\eta\right)dx.$$

Using the fact that when $T_s = T_f = 0$, we have that $(1+\beta\theta)(1-\theta)x^\eta = Wx$, we get that when $T_f = T_s = 0$, the equation above simplifies to

$$(1-\beta(1-\theta))\,dT_f + dT_s = (1-\eta)\,Wdx, \tag{A.18}$$

so that

$$\left.\frac{\partial x}{\partial T_f}\right|_{T_f=T_s=0} = \frac{1}{(1-\eta)W}, \qquad \left.\frac{\partial x}{\partial T_s}\right|_{T_f=T_s=0} = \frac{1-\beta(1-\theta)}{(1-\eta)W}$$

□

A.3 Data appendix

In this section, we detail the procedure we take to construct the balanced panel dataset we use in our analysis. We then present some descriptive statistics.

The raw unbalanced dataset between 2007–2015 contains 15,125,421 firm-year observations. We drop firms with consolidation codes "NA" (no financial data available), "LF" (limited financial data available), "C1" (consolidated statement with unconsolidated companion), and "C2" (consolidated statement with no unconsolidated companion). Leaving us with unconsolidated statements only - resulting in 8,418,833 firm-year observations.[42]

We then drop observations where total assets, accounts receivable, accounts payable, operating revenues, or sales are missing. Additionally, we drop observations with negative values for one of the following variables: total assets, employment, sales, wage bill, accounts receivable, accounts payable, short-term bank loans, long-term debt, depreciation, cash holdings, or total inventories. Finally, we drop observations where the year of operation is earlier than the date of incorporation of the firm. These steps leave us with 8,107,403 firm-year observations.

We drop financial firms (136,677 obs.), drop firms with less than 10,000 in total assets or annual sales (30,226 obs), and keep only firms with an "active" status (dropping 1,393,729 observations). We then keep firms with observations in all years so that the panel is balanced. The balanced panel contains 2,447,163 observations.

We construct a variable for intermediate inputs which subtracts the sum of operating profits, wage bill, and depraciation, from sales. For each observation we compute the ratio between accounts payable, accounts receivable, and intermediate inputs to sales. We winsorize the top and bottom 1%. Finally, we drop the top 1% and bottom 1% growing firms in terms of log sales in the data.[43]

The final dataset contains a balanced panel of 243,553 firms over 2007–2015, resulting in 2,191,977 observations. Table A-1 displays descriptive statistics using the final balanced sample of firms.

A.4 Numerical Algorithm

In this section, we lay out the numerical algorithm we use to solve the model. Our approach relies on policy function iteration combined with an approximate law-of-motion similar to

[42] We additionally drop 1,908 duplicate observations.

[43] We also drop 1,305 firm-year observations in the refinery sector as most purchases of refinery products are imported.

Table A-1: Descriptive statistics at the firm-level

Variable	Mean	P10	P25	P50	P75	P90	% of firms with data
Total assets (Millions)	8,022,889	139,528	329,347	903,717	2,683,331	7,910,160	100%
Sales	6,106,199	85,877	229,836	711,568	2,250,524	6,962,928	100%
Accounts payable	1,117,917	0	6,971	87,004	421,124	1,494,970	100%
Accounts receivable	1,380,116	0	6,227	120,835	571,589	2,010,485	100%
Short-term bank loans	677,022	0	0	6,767	160,416	779,963	100%
Intermediate inputs	6,332,176	111,181	265,464	734,417	2,280,475	7,030,696	78%

Krusell-Smith. We start by defining some notation. We then show how to we solve for the partial equilibrium, where aggregate levels are taken as given. Finally, we show how we solve for the full general equilibrium.

We show how to solve for the general case with corporate subsidies. The benchmark model is a special case when such subsidies are set to zero. To make it easy to see where these subsidies enter the equations, we color all such places in blue.

A.4.1 Notation

Before describing the numerical algorithm in details, it is useful to define some notation.The revenue of final-good producers in sector i is denoted by

$$rev_i(x) = C(\theta,\Omega)^{\frac{1}{\gamma}} N_i^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}} x^{\eta\frac{\gamma-1}{\gamma}}.$$

The first-best labor input in sector i is denoted by

$$x^{\text{fb}} = \left[\frac{\gamma-1}{\gamma}\eta\frac{C(\theta,\Omega)^{\frac{1}{\gamma}}}{w(\theta,\Omega)} N_i^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}-1}\right]^{\frac{\gamma}{(1-\eta)\gamma+\eta}}.$$

The promise keeping constraint is

$$\tilde{J}(\theta) = A_i rev_i - \frac{1}{N_i}p^s - \frac{1}{N_i}p^{tc} + \beta\mathbb{E}_\theta[\tilde{J}'(\theta')],$$

where

$$A_i \equiv 1 - \left(\frac{N_i-1}{N_i}\right)^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}.$$

Next, denote by $x^{\text{cons}}(\tilde{J}_\theta, \theta)$ the solution to $rev_i(x) = \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\theta(1-\delta(1-\pi_i))}{N_i}}$. Namely,

$$x^{\text{cons}}(\tilde{J}_\theta, \theta) = \left[\frac{\tilde{J}_\theta + T_f(\theta)}{\left(A_i - \frac{1-\theta(1-\delta(1-\pi_i))}{N_i}\right) C(\theta, \Omega)^{\frac{1}{\gamma}} N_i^{\eta \frac{\sigma}{\sigma-1} \frac{\gamma-1}{\gamma}}} \right]^{\frac{\gamma}{\eta(\gamma-1)}}$$

Finally, to ease notation, we introduce

$$\tilde{\theta}_i = \theta(1 - \delta(1 - \pi_i)),$$

where $\tilde{\theta}'_i$ is the equation above but with θ' instead of θ.

A.4.2 Partial Equilibrium

We start by solving the behavior of suppliers in a single sector taking the aggregate values $C(\theta, \Omega)$ and $w(\theta, \Omega)$ as given (simply denoted by C and w for current values, and C', and w' for next period's values). We assume that θ can take one of two values, θ_L and θ_H. The numerical algorithm uses policy function iteration to find the policy function in equilibrium. The policy function maps the current state of the economy, which includes both the aggregate state of financial frictions θ as well as the promise keeping value $\tilde{J}$, into two promise keeping values in the next period, one for each degree of financial frictions, θ'. The policy function is denoted by

$$\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta),$$

which denotes the promise keeping value the supplier must deliver in state θ', given that the current state is $\{\tilde{J}_\theta, \theta\}$.

Our policy function iteration proceeds in three steps:

1. Given a guess for the policy function, and the current state $\{\tilde{J}_\theta, \theta\}$, we compute future labor inputs x' for every θ' as a function of the current state $\{\tilde{J}_\theta, \theta\}$ and current Lagrange multipliers on the borrowing constraint of the supplier (ι) and the feasibility requirement (ρ).
2. Using the implied $x'(\theta'; \tilde{J}_\theta, \theta, \iota, \rho)$ function, we update the policy function.
3. If the implied policy function is close to the guessed one, we have found the policy function in equilibrium. Otherwise, we update our guess for the policy function using the implied policy function, and repeat from step (1).

Below, we provide a detailed explanation for how each of the two steps are performed.

The state space grid is over $\{\tilde{J}_\theta, \theta\}$, where upper bound for $\tilde{J}_\theta$ is given by $\left(A_i - \frac{1-\theta'}{N_i}\right) rev_i^{fb}$ and the lower bound is close to zero.

A.4.2.1 Calculating x' given the policy function and current Lagrange multipliers

The optimality condition for x' (A.4) can be written as follows

$$\frac{\gamma-1}{\gamma}\eta\left(C'\right)^{\frac{1}{\gamma}} N_i^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}-1}\left(x'\right)^{\eta\frac{\gamma-1}{\gamma}-1} = \frac{w'(1+\iota')}{\left(1-\tilde{\theta}_i'\right)\mu'+\rho'+\lambda'}. \tag{A.19}$$

In this section, we show how we can use the Lagrange multipliers ι and ρ, together with the state variables $\{\tilde{J}_\theta, \theta\}$, and the policy function $\tilde{J}'\left(\theta' \mid \tilde{J}_\theta, \theta\right)$, to compute x'. That is, we construct the function $x'(\theta'; \tilde{J}_\theta, \theta, \iota, \rho)$.

To find x', we will need to use the promise keeping in two periods, J''. To do so we apply the policy function $\tilde{J}'(\cdot)$ twice, starting from the state $\{\tilde{J}_\theta, \theta\}$. In particular, given the current state $\{\tilde{J}_\theta, \theta\}$, as well as θ', the expected promise keeping in two periods is given by

$$\mathbb{E}_{\theta'}[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)] \equiv \mathbb{E}_{\theta'}[\tilde{J}'(\theta'' \mid \tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta), \theta')]. \tag{A.20}$$

Note that when we apply the policy function twice, the second time we apply it can potentially be done for a point J' which is not a grid point. We use Chebyshev approximation to apply the policy function to points which lie between points on the grid.

We start by deriving an equation connecting the current Lagrange multipliers ι and ρ, with future ones (μ', ρ', ι'). From optimality condition (A.2) we have

$$\mu' + \rho' - \iota' = 1 - \lambda',$$

Using equation (A.7), together with optimality condition A.3 from the current period, we obtain

$$\rho - (1-\theta)\iota = \mu' + \rho' - \iota'. \tag{A.21}$$

Therefore, by knowing ι and ρ in the current period, we obtain the sum of $\mu' + \rho' - \iota'$. We split the derivation of x' into two cases, depending on the sign of $\rho - (1-\theta)\iota$.

<u>Case A</u>: $\rho - (1-\theta)\iota \geq 0$.

Suppose that the borrowing constraint of the supplier and the feasibility constraint do not bind ($\iota' = \rho' = 0$). In this case we have $\mu' = \rho - (1-\theta)\iota$. And from optimality condition

(A.2), we have $\lambda' = 1 - \mu'$. The optimality condition for x' (A.19) is then

$$\frac{\gamma-1}{\gamma}\eta\left(C'\right)^{\frac{1}{\gamma}} N_i^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}-1}\left(x'\right)^{\eta\frac{\gamma-1}{\gamma}-1} = \frac{w'}{1-\tilde{\theta}_i'\mu'}.$$

Plugging in the value of μ' and the aggregate variables, we obtain x'. We now need to check whether the borrowing constraint of the supplier and the feasibility constraint are satisfied. [44]

$$w'x' - \frac{1-\tilde{\theta}_i'}{N_i} rev_i(x') \leq (1-\theta')\beta\mathbb{E}_{\theta'}[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)] + T_s(\theta') + T_f(\theta'), \tag{A.22}$$

$$\tilde{\theta}_i' rev_i(x') \geq N_i\beta\mathbb{E}_{\theta'}[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)]. \tag{A.23}$$

Note that final-good producers obtain T_f per supplier. So they receive $N_i T_f$ in total.

In case both equations (A.22)–(A.23) above are satisfied, we have found $x'(\theta'; \tilde{J}_\theta, \theta, \iota, \rho)$. Otherwise, we proceed as follows:

1. If both constraints (A.22)–(A.23) are violated then $\mu' > 0$, $\rho' > 0$ and $\iota' > 0$. In that case, both the borrowing constraint and the feasibility constraint binds so combining equations (A.22)–(A.23) with equality we obtain

$$w'x' = \frac{1-\theta'\tilde{\theta}_i'}{N_i} rev_i(x') + T_f(\theta') + T_s(\theta'), \tag{A.24}$$

   This equation pins down the level of x', and we obtain $x'(\theta'; \tilde{J}_\theta, \theta, \iota, \rho)$. Note that when $T_f = T_s = 0$, we can derive an explicit formula for x.

2. If only the supplier's borrowing constraint (A.22) is violated, then $\mu' > 0$ and $\iota' > 0$. We guess that the feasibility constraint is not binding so that $\rho' = 0$. From equation (A.21) we have

$$\mu' - \iota' = \rho - (1-\theta)\iota \geq 0.$$

   From optimality conditions (A.2)–(A.3), we have that $\mu' = \theta'\iota' + \gamma'$. So for $\mu' - \iota' \geq 0$, it must be that $\gamma' > 0$. So the borrowing constraints of both the supplier and the final-good producer are binding, as well as the trade-credit constraint. Combining all three equations we obtain

$$w'x' - \frac{1-\tilde{\theta}_i'}{N_i} rev_i(x') = (1-\theta')\beta\mathbb{E}_{\theta'}[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)] + T_f(\theta') + T_s(\theta'). \tag{A.25}$$

[44] When using $rev_i(x')$, aggregate consumption in the revenue function is C'.

This equation pins down x'. We now need to verify whether the feasibility constraint is indeed satisfied. We check whether

$$\tilde{\theta}_i' rev_i(x') \geq N_i \beta \mathbb{E}_{\theta'}[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)].$$

If the equation above holds, then we found $x'(\theta'; \tilde{J}_\theta, \theta, \iota, \rho)$. Otherwise, also $\rho' > 0$ and we obtain x' from step 1.

3. If only the feasibility constraint (A.23) is violated, then $\mu' > 0$ and $\rho' > 0$. We guess that the supplier borrowing constraint is not binding, $\iota' = 0$. From optimality conditions (A.2)–(A.3), we have that $\mu' = \theta'\iota' + \gamma'$, so that $\gamma' = \mu' > 0$. So we have that the borrowing constraint of the final good producer is binding as well as the trade credit constraint and the feasibility constraint. We combine the three constraints to obtain

$$\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x') = \tilde{J}'\left(\theta'; \tilde{J}_\theta, \theta\right) + T_f(\theta'), \tag{A.26}$$

which pins down the value of x'. We need to check whether the borrowing constraint of the supplier is satisfied:

$$w'x' \leq \frac{1-\theta'\tilde{\theta}_i'}{N_i} rev_i(x') + T_f(\theta') + T_s(\theta').$$

If the equation above holds, we have found $x'(\theta'; \tilde{J}_\theta, \theta, \iota, \rho)$. Otherwise, also $\iota' > 0$ and we obtain x' from step 1.

Case B: $\rho - (1-\theta)\iota < 0$

From equation (A.21) it follows that $\iota' > 0$, as both μ' and ρ' are non-negative. From optimality conditions (A.2)–(A.3), we have that $\mu' = \theta'\iota' + \gamma'$. Since $\gamma' \geq 0$, we have that $\mu' > 0$. We proceed in the following steps.

1. Suppose $\rho' = 0$. We solve for two cases. First, assuming that $\gamma' = 0$. And if the solution in that case violates the trade credit constraint, we move to the second case where $\gamma' > 0$.

   (a) Suppose $\gamma' = 0$. In this case, $\mu' = \theta'\iota'$. Plugging into (A.21), we have

   $$\iota' = \frac{(1-\theta)\iota - \rho}{1-\theta'}.$$

   So we know both the values of ι' and $\mu' = \theta'\iota'$. From optimality condition (A.2), we obtain $\lambda' = 1 + \iota' - \mu'$. Plugging into equation (A.19), we then obtain x'. We need to check whether the trade credit constraint is satisfied. We use the supplier's borrowing constraint to obtain

$$p_j^{\mathrm{tc}\prime} = \frac{1}{1-\theta'}\left[w'x' - \frac{1-\tilde{\theta}_i'}{N_i} rev_i(x') - T_f(\theta') - T_s(\theta')\right].$$

So for the trade credit constraint to hold it must be that

$$w'x' - \frac{1-\tilde{\theta}_i'}{N_i} rev_i(x') - T_f(\theta') - T_s(\theta') \leq (1-\theta')\beta \mathbb{E}_{\theta'}[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)]. \qquad \text{(A.27)}$$

If the condition above holds, then we have a candidate for x' and only need to confirm $\rho' = 0$, which we do after the next subcase. Otherwise, we move to the next subcase where $\gamma' > 0$.

(b) In this subcase $\rho' = 0$, while $\iota' > 0$, $\gamma' > 0$, and $\mu' > 0$. That is, the borrowing constraints of both the supplier and final good producer hold with equality, as well as the trade credit constraint. Combining all three equations we obtain

$$w'x' - \frac{1-\tilde{\theta}_i'}{N_i} rev_i(x') = (1-\theta')\beta \mathbb{E}_{\theta'}[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)] + T_f(\theta') + T_s(\theta'). \qquad \text{(A.28)}$$

The equation above uniquely pins down x'. In this case, we have

$$p_j^{\mathrm{tc}\prime} = \beta \mathbb{E}_{\theta'}[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)].$$

We move to check whether the feasibility constraint holds so that $\rho' = 0$.

After obtaining x' and $p_j^{tc\prime}$ from case (a) and (b), we need to verify the feasibility constraint is not violated. We check the following condition

$$p_j^{tc\prime} \leq \frac{\tilde{\theta}_i'}{N_i} rev_i(x'). \qquad \text{(A.29)}$$

If this condition holds, we have found $x'(\theta'; \tilde{J}_\theta, \theta, \iota, \rho)$. Otherwise, we conclude that $\rho' > 0$ and move to case 2 below.

2. In this case, we have that ι', μ' and ρ' are all strictly positive. That is, the borrowing constraints of both the supplier and final-good producer hold with equality as well

as the feasibility constraint. Combining all three equations we obtain equation (A.24):

$$w'x' = \frac{1-\theta'\tilde{\theta}_i'}{N_i} rev_i(x') + T_f(\theta') + T_s(\theta').$$

This equation uniquely pins down x', and we have found $x'(\theta'; \tilde{J}_\theta, \theta, \iota, \rho)$.

A.4.2.2 Updating the guess of the policy function

In the previous section we have derived the function $x'(\theta'; \tilde{J}_\theta, \theta, \iota, \rho)$, which implicitly takes into account also the guess of the policy function. In this section, we use this function to find the equilibrium allocations in the current period, and to update the guess for the policy function.

Given x', the guess for the policy function is

$$\tilde{J}'(\theta') = \left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x') - T_f(\theta'), \tag{A.30}$$

which effectively assumes that the borrowing constraint of the final-good producer as well as the trade credit constraint hold with equality.

We proceed in cases, where each corresponds to a different set of Lagrange multipliers being strictly positive. Overall there are six cases. Case (I) considers the case in which all current multipliers are 0. Case (II) considers the case where μ and γ are strictly positive, while $\rho = \iota = 0$. Case (III) considers the case in which only $\iota = 0$. Case (IV) considers the case in which μ and ι are strictly positive, while $\gamma = \rho = 0$. Case (V) considers the case in which only $\rho = 0$. Finally, case (VI) assumes that μ, ι, and ρ are all strictly positive, while $\gamma \geq 0$. Note that in general there could be 16 options for which Lagrange multipliers are strictly positive, but using the optimality conditions, we narrow it down to six different combinations which can occur in equilibrium.

I) $\mu = \gamma = \rho = \iota = 0$. This case applies when the following two conditions hold:

$$wx^{\text{cons}}(\tilde{J}_\theta, \theta) \leq \frac{1-\tilde{\theta}_i}{N_i}\frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + (1-\theta)\beta\mathbb{E}_\theta\left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0) - T_f(\theta')\right] + T_s(\theta) + T_f(\theta),$$

and

$$\tilde{J}(\theta) + T_f(\theta) \geq \left(A_i - \frac{1-\tilde{\theta}_i}{N_i}\right) rev_i^{\text{fb}}.$$

In this case the solution is:

$$
\begin{aligned}
x &= x^{fb}, \\
p_j^s &= \frac{1}{N_i}(1-\tilde{\theta}_i)rev_i^{fb} + T_f(\theta), \\
p_j^{tc} &= \beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0)) - T_f(\theta') \right], \\
\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta) &= \left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0)) - T_f(\theta').
\end{aligned}
$$

II) $\mu = \gamma > 0$, and $\rho = \iota = 0$. This case applies when the following three conditions hold:

$$
\begin{aligned}
& wx^{\text{cons}}(\tilde{J}_\theta, \theta) \leq \frac{1-\tilde{\theta}_i}{N_i} \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + (1-\theta)\beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0) - T_f(\theta') \right] + T_s(\theta) + T_f(\theta), \\
& \tilde{J}(\theta) + T_f(\theta) < \left(A_i - \frac{1-\tilde{\theta}_i}{N_i} \right) rev_i^{\text{fb}}, \\
& \tilde{J}(\theta) + T_f(\theta) \geq \beta N_i \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0) - T_f(\theta') \right] \frac{A_i - \frac{1-\tilde{\theta}_i}{N_i}}{\tilde{\theta}_i}.
\end{aligned}
$$

The solution in this case is:

$$
\begin{aligned}
x &= x^{cons}(\tilde{J}_\theta, \theta), \\
p_j^s &= \frac{1}{N_i}(1-\tilde{\theta}_i) \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + T_f(\theta), \\
p_j^{tc} &= \frac{1}{N_i} \beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0)) - T_f(\theta') \right], \\
\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta) &= \left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0)) - T_f(\theta').
\end{aligned}
$$

III) $\mu = \gamma > 0$, $\rho > 0$, and $\iota = 0$. This case applies when the following three conditions hold:

$$wx^{\text{cons}} \le \frac{1-\tilde{\theta}_i}{N_i}\frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + (1-\theta)\left[\frac{\tilde{\theta}_i\left(\tilde{J}(\theta) + T_f(\theta)\right)}{N_i A_i - (1-\tilde{\theta}_i)}\right] + T_s(\theta) + T_f(\theta),$$

$$\tilde{J}(\theta) + T_f(\theta) < \beta N_i \mathbb{E}_\theta\left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0)) - T_f(\theta')\right]\frac{A_i - \frac{1-\tilde{\theta}_i}{N_i}}{\tilde{\theta}_i},$$

$$\tilde{J}(\theta) + T_f(\theta) \ge \beta N_i \mathbb{E}_\theta\left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 1)) - T_f(\theta')\right]\frac{A_i - \frac{1-\tilde{\theta}_i}{N_i}}{\tilde{\theta}_i}.$$

The solution in this case is:

$$x = x^{cons}(\tilde{J}_\theta, \theta),$$

$$p_j^s = \frac{1}{N_i}(1-\tilde{\theta}_i)\frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + T_f(\theta),$$

$$p_j^{tc} = \beta \mathbb{E}_\theta\left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, \bar{\rho})) - T_f(\theta')\right],$$

$$\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta) = \left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, \bar{\rho})) - T_f(\theta'),$$

where $\bar{\rho}$ is defined implicitly by solving the following equation:

$$\beta N_i \mathbb{E}_\theta\left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, \bar{\rho})) - T_f(\theta')\right] = \tilde{\theta}_i \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}}.$$

If $\{\tilde{J}_\theta, \theta\}$ is such that the conditions to cases (I)–(III) do not hold, it must be that $\iota > 0$. This implies $\mu > 0$. We first suppose the other two constraints are slack (case IV), then if these constraints are violated, we move to case (V) and to case (VI).

IV) $\mu > 0$, $\iota > 0$. We conjecture $\rho = \gamma = 0$. When $\gamma = 0$, we have that $\mu = \theta\iota$ so that the optimality condition for x can be written as follows:

$$\left\{1 + \left[1 - \theta\tilde{\theta}_i\right]\iota\right\}\frac{\gamma - 1}{\gamma}\eta C^{\frac{1}{\gamma}} N_i^{\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}-1} x^{\eta\frac{\gamma-1}{\gamma}-1} = w(1+\iota).$$

From the equation above, we can solve for x as a function of ι:

$$x(\iota) = \left[\frac{1 + \left[1 - \theta\tilde{\theta}_i\right]\iota}{w(1+\iota)} \quad \frac{\eta C^{\frac{1}{\gamma}}}{N_i^{1-\eta\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}}\frac{\gamma-1}{\gamma}\right]^{\frac{\gamma}{(1-\eta)\gamma+\eta}}.$$

Since both the final-good producer's and supplier's borrowing constraints bind ($\iota > 0$

and $\mu > 0$), we can combine the two to obtain the level of trade credit as a function of x:

$$p_j^{tc} = \frac{wx}{1-\theta} - \frac{1}{N_i}\frac{1-\tilde{\theta}_i}{1-\theta} rev_i(x) - \frac{T_f(\theta)+T_s(\theta)}{1-\theta}.$$

Then, using the promise keeping constraint, we have

$$\tilde{J}_\theta = \left(A_i + \frac{\theta(1-\tilde{\theta}_i)}{N_i(1-\theta)}\right) rev_i(x) - \frac{wx}{1-\theta} + \frac{\theta T_f(\theta)+T_s(\theta)}{1-\theta} + \beta\mathbb{E}_\theta[\tilde{J}'(\theta')].$$

We then find $\bar{\iota}$ so that the promise keeping constraint holds with equality:

$$\tilde{J}_\theta = \left(A_i + \frac{\theta(1-\tilde{\theta}_i)}{N_i(1-\theta)}\right) rev_i(x(\bar{\iota})) - \frac{wx(\bar{\iota})}{1-\theta} + \frac{\theta T_f(\theta)+T_s(\theta)}{1-\theta} + \beta\mathbb{E}_\theta\left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta';\tilde{J}_\theta,\theta,\bar{\iota},0)) - T_f(\theta')\right].$$

The solution in this case is:

$$\begin{aligned}
x &= x(\bar{\iota}),\\
p_j^s &= \frac{1-\tilde{\theta}_i}{N_i} rev_i(x(\bar{\iota})) + T_f(\theta),\\
p_j^{tc} &= \frac{wx(\bar{\iota})}{1-\theta} - \frac{1}{N_i}\frac{1-\tilde{\theta}_i}{1-\theta} rev_i(x(\bar{\iota})) - \frac{T_f(\theta)+T_s(\theta)}{1-\theta},\\
\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta) &= \left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta';\tilde{J}_\theta,\theta,\bar{\iota},0)) - T_f(\theta').
\end{aligned}$$

Since this case assumed two Lagrange multipliers are zero, to verify the solution there are two conditions we need to verify:

$$\tilde{J}_\theta \geq \left(A_i - \frac{1-\tilde{\theta}_i}{N_i}\right) rev_i(x(\bar{\iota})) - T_f(\theta), \tag{A.31}$$

$$wx(\bar{\iota}) \leq \left(1-\theta+\theta(1-\tilde{\theta}_i)\right) rev_i(x(\bar{\iota})) + T_f(\theta) + T_s(\theta). \tag{A.32}$$

Equation (A.31) ensures that the trade credit constraint is satisfied, while equation (A.32) ensures the feasibility constraint is satisfied. If both conditions hold, then we have the solution above is valid. If condition (A.32) is satisfied but condition (A.31) is violated, then $\gamma > 0$ and we move to case (V). If condition (A.32) is violated, we move to case (VI).

V) $\mu > 0$, $\iota > 0$, $\gamma > 0$, and we conjecture $\rho = 0$. Since the borrowing constraint of the final-good producer binds and the trade credit constraint binds we have

$$x = x^{\text{cons}}(\tilde{J}_\theta, \theta),$$
$$rev_i(x) = \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}}.$$

Since the supplier's borrowing constraint binds as well ($\iota > 0$), we also have that

$$wx = \frac{1-\theta}{N_i} rev_i(x) + T_f(\theta) + T_s(\theta) + (1-\theta)\beta \mathbb{E}_\theta[\tilde{J}'(\theta')].$$

Therefore, we can find ι^* from the following equation

$$wx^{\text{cons}} = \frac{1-\tilde{\theta}_i}{N_i} \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + T_f(\theta) + T_s(\theta)$$
$$+ (1-\theta)\beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, \iota^*, 0)) - T_f(\theta') \right]$$

We need to verify that the feasibility constraint is satisfied, which boils down to the following condition:

$$\beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, \iota^*, 0)) - T_f(\theta') \right] \leq \frac{\tilde{\theta}_i}{N_i} \frac{\tilde{J}(\theta) + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}}.$$

If this condition is violated, we move to case (VI). Otherwise, the solution is:

$$x = x^{\text{cons}}(\tilde{J}_\theta, \theta),$$
$$p_j^s = \frac{1}{N_i}(1-\tilde{\theta}_i) \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + T_f(\theta),$$
$$p_j^{tc} = \beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, \iota^*, 0)) - T_f(\theta') \right],$$
$$\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta) = \left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, \iota^*, 0)) - T_f(\theta').$$

VI) $\mu > 0$, $\iota > 0$, $\rho > 0$, and $\gamma \geq 0$. When the borrowing constraints of both the final-good producer and the supplier bind, and the feasibility constraint is also binding, we obtain:

$$
\begin{aligned}
p_j^s &= \frac{(1-\tilde{\theta}_i)}{N_i} rev_i(x) + T_f(\theta), \\
p^{tc} &= \frac{\tilde{\theta}_i}{N_i} rev_i(x), \\
wx &= \frac{1-\theta\tilde{\theta}_i}{N_i} rev_i(x) + T_f(\theta) + T_s(\theta).
\end{aligned}
$$

The final equation pins down implicitly the level of x, which we denote by $\bar{x}$. The promise keeping constraint is then

$$
\tilde{J}(\theta) = \left(A_i - \frac{1}{N_i}\right) rev_i(\bar{x}) - T_f(\theta) + \beta \mathbb{E}_\theta \left[\tilde{J}'(\theta')\right].
$$

Finally, we take advantage of the fact that the function $x'(\cdot)$ depends on the value of $\rho - (1-\theta)\iota$, regardless of the individual values of ρ and ι. Let $\zeta \equiv \rho - (1-\theta)\iota$. Then, the value of ζ is given by the solution to the following equation:

$$
\tilde{J}(\theta) = \left(A_i - \frac{1}{N_i}\right) rev_i(x) - T_f(\theta) + \beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, \zeta)) - T_f(\theta')\right].
$$

The solution in this case is:

$$
\begin{aligned}
x &= \bar{x}, \\
p_j^s &= \frac{(1-\tilde{\theta}_i)}{N_i} rev_i(x) + T_f(\theta), \\
p_j^{tc} &= \frac{\tilde{\theta}_i}{N_i} rev_i(x), \\
\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta) &= \left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, \zeta)) - T_f(\theta').
\end{aligned}
$$

A.4.3 General equilibrium

The previous section detailed our algorithm for solving the policy function of suppliers in an industry given current and future aggregate variables, C_t and w_t. In this section, we explain how we solve the full general equilibrium model with aggregate fluctuations.

Firms in the model need to form beliefs regarding future levels of consumption and the real wage. The full state variable in the economy contains the degree of financial frictions θ_t as well as the distribution of promise keeping values across all sectors $\{J_{it}\}_i$. As our model contains 58 sectors, the curse of dimensionality prevents us from solving the model as a function of all J's. Instead, we conjecture that the level of consumption as well as the real wage follow an AR(1) process in logs, where the AR(1) coefficients depend on the degree of financial frictions. That is,

$$\ln C_t = (1-\rho_c(\theta_t))\mu_c(\theta_t) + \rho_c(\theta_t)\ln C_{t-1},$$
$$\ln w_t = (1-\rho_w(\theta_t))\mu_w(\theta_t) + \rho_w(\theta_t)\ln w_{t-1},$$

where $\rho_c(\theta_t)$, $\mu_c(\theta_t)$, $\rho_w(\theta_t)$, and $\mu_w(\theta_t)$ are a function of θ. Since we have two levels of θ, this leads to 8 unknowns. We denote the vector of these 8 state variables as $\vec{\zeta}$. We show below that this formulation yields an accurate approximation for the law of motion of both C_t and w_t.

Given the laws of motion for aggregate consumption and the real wage, suppliers only need to know the level of consumption and real wage in order to form beliefs on the future. Thus, the policy function can be written as $\tilde{J}'(\theta'; \tilde{J}_\theta, \theta, \ln C^L, \ln w^L)$, where the value denotes the promised surplus in the next period if the degree of financial frictions is θ', given the current promised surplus ($\tilde{J}_\theta$), the current degree of financial frictions (θ), lagged log aggregate consumption ($\ln C^L$), and the lagged log-level of the real wage ($\ln w^L$).

We adapt the partial equilibrium algorithm to solve for the policy function in general equilibrium, given the laws of motion ($\vec{\zeta}$). Instead of the state space being $\{\tilde{J}_\theta, \theta\}$, the state space is now given by $\{\tilde{J}_\theta, \theta, \ln C^L, \ln w^L\}$. Solving the policy function then follows exactly the same steps as described in the previous section, with two adjustments. First, the future levels of aggregate consumption and real wage vary with the future level of θ according to their laws of motion. Second, the expected future promise keeping value in two periods (A.20) takes into account the laws of motion:

$$\begin{aligned}&\mathbb{E}_{\theta'}[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta, \ln C^L, \ln w^L)] \equiv \\ &\quad \mathbb{E}_{\theta'}[\tilde{J}'(\theta'' \mid \tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta, \ln C^L, \ln w^L), \theta', (1-\rho_c(\theta))\mu_c(\theta) + \rho_c(\theta)\ln C^L, (1-\rho_w(\theta))\mu_w(\theta) + \rho_w(\theta)\ln w^L)].\end{aligned}$$

We proceed as follows:

1. Start with a guess for $\vec{\zeta}$.
2. Given $\vec{\zeta}$, solve the policy function for each of the 58 sectors in the economy independently.
3. Simulate θ_t for T periods using the transition matrix.[45] Use $\vec{\zeta}$ to obtain C_t and w_t,

[45]We set $T = 5,000$ when solving the model.

starting with the first value being the steady state one when $\theta = \theta_L$.

4. Using the policy functions together with the sequence $\{\theta_t, C_t, w_t\}$, find the employment of suppliers to each sector and the level of output produced by each sector. Denote the implied output levels by $y_{it}(\vec{\zeta})$ and total employment of suppliers to sector i by $x_{it}(\vec{\zeta})$.
5. Use the definition of aggregate consumption as well as the optimality condition of households to obtain the implied levels of aggregate consumption and the real wage:

$$C_t(\vec{\zeta}) = \left[\frac{1}{58}\sum_i y_{it}(\vec{\zeta})^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}},$$

$$w_t(\vec{\zeta}) = \chi\left[\frac{1}{58}\sum_i N_i x_{it}(\vec{\zeta})\right]^{\frac{1}{\psi}}.$$

6. Regress $\ln C_t(\vec{\zeta})$ on $\ln C_{t-1}(\vec{\zeta}) \times \mathbb{1}(\theta_t = \theta_L)$, $\ln C_{t-1}(\vec{\zeta}) \times \mathbb{1}(\theta_t = \theta_H)$, $\mathbb{1}(\theta_t = \theta_L)$, and $\mathbb{1}(\theta_t = \theta_H)$ (no constant). Run a similar regression for $\ln w_t(\vec{\zeta})$. Denote the regression coefficients for these implied laws-of-motion as $\bar{\zeta}$.
7. If $\bar{\zeta}$ is sufficiently close to $\vec{\zeta}$, we have solved the model. Otherwise, update the law-of-motion coefficients $\vec{\zeta}$ to be a convex combination of the current guess $\vec{\zeta}$ and the implied ones $\bar{\zeta}$. In particular, set the new guess to be $0.5\vec{\zeta} + 0.5\bar{\zeta}$. Then repeat from step (2).

For our benchmark specification, the $\mathbb{R}^2$ for the regression of the law-of-motion for log-consumption is 0.99999975 and the $\mathbb{R}^2$ for the regression of the law-of-motion for log-wage is 0.99999974. That is, the approximation is very accurate.